COLORADO SCHOOL OF MINES
ELECTRICAL ENGINEERING & COMPUTER SCIENCE DEPARTMENT

EENG 382
Engineering Circuit Analysis
Exam #3
November 25, 2013

Directions:

You may use a one-sided 8.5”x11” sheet with formulas, notes, etc., which you must turn in with your exam. Be sure to put you name on your formula sheet. No other materials are allowed (except for a calculator, pencil, and eraser).

Name: KEY

SCORES

MC:______/20 PROB #1:_______/20 PROB #2:_______/20

TOTAL:_________/60

Multiple Choice Responses

Instructions: Choose the BEST answer of those given. Please indicate your answer by DARKENING the response that you have chosen. Please make your selection obvious.

	1.	a	b	c	d	e	6.	a	b	c	d	e
	2.	a	b	c	d	e	7.	a	b	c	d	e
	3.	a	b	c	d	e	8.	a	b	c	d	e
NO ANSWER 2+1=3	4.	a	b	c	d	e	9.	a	b	c	d	e
	5.	a	b	c	d	e	10.	a	b	c	d	e

Multiple Choice Section: Each multiple choice question is worth 2 points. Please circle the answer you have selected and carefully record your response on the cover sheet of the exam. There is no partial credit in this section.

Question 1. What is the Fourier Transform of $f(t) = 10Ve^{-|t|/\tau}\cos(\omega_o t)$?

UNITS ~~a)~~ $F(\omega) = 10V\,\tau\left[\frac{1}{\frac{1}{\tau^2}+(\omega+\omega_o)^2}+\frac{1}{\frac{1}{\tau^2}+(\omega-\omega_o)^2}\right]$

b) $F(\omega) = \frac{10V}{\tau}\left[\frac{1}{\frac{1}{\tau^2}+(\omega+\omega_o)^2}+\frac{1}{\frac{1}{\tau^2}+(\omega-\omega_o)^2}\right]$

UNITS ~~c)~~ $F(\omega) = \frac{10V}{\tau}\left[\frac{1}{1+\tau^2(\omega+\omega_o)^2}+\frac{1}{1+\tau^2(\omega-\omega_o)^2}\right]$

UNITS ~~d)~~ $F(\omega) = 10V\left[\frac{\tau^2}{\frac{1}{\tau^2}+(\omega+\omega_o)^2}+\frac{\tau^2}{\frac{1}{\tau^2}+(\omega-\omega_o)^2}\right]$

UNITS ~~e)~~ $F(\omega) = 10V\left[\frac{1}{\frac{1}{\tau^2}+(\omega+\omega_o)^2}+\frac{1}{\frac{1}{\tau^2}+(\omega-\omega_o)^2}\right]$

$\mathcal{F}\{g(t)\cos(\omega_o t)\} = \frac{1}{2}\left[G(\omega-\omega_o)+G(\omega+\omega_o)\right]$

$g(t) = e^{-\frac{|t|}{\tau}} \Leftrightarrow G(\omega) = \frac{2}{\tau}\left(\frac{1}{\left(\frac{1}{\tau}\right)^2+\omega^2}\right)$

$F(\omega) = 10V\cdot\frac{1}{2}\cdot\frac{2}{\tau}\left[\frac{1}{\left(\frac{1}{\tau}\right)^2+(\omega-\omega_o)^2}+\frac{1}{\left(\frac{1}{\tau}\right)^2+(\omega-\omega_o)^2}\right]$

Question 2. What are the units on the Fourier Transform of a voltage waveform?

a) $V/\sqrt{Hz}$

$\mathcal{F}\{v(t)\} = \int_{-\infty}^{\infty} v(t)\,e^{-j\omega t}\,dt \Rightarrow$ UNITS OF $V\cdot sec$

b) V

c) V/sec

d) $V\,sec$

e) $V\,Hz$

Question 3. The cosine voltage coefficients vanish for which of the following waveforms?

ANY WAVEFORM WITH ODD SYMMETRY

a) Any waveform with even symmetry (EVEN)

b) A triangle wave that peaks at t=0s. (EVEN)

c) A square wave that has a rising edge at t=0s. (ODD)

d) Any waveform that has an average value of zero. (MAY BE EVEN)

e) Any function possessing half-wave symmetry. (MAY BE EVEN)

Question 4. Given $f(t) = 8V\sin(\omega_o t) - 6V\cos(\omega_o t)$, which of the following is equivalent?

| OFF BY |

106.2^o a) $f(t) = 10V\cos(\omega_o t + 126.9^o)$

90^o b) $f(t) = 10V\cos(\omega_o t - 36.9^o)$

83.1^o c) $f(t) = 10V\sin(\omega_o t - 120^o)$

73.8^o d) $f(t) = 10V\sin(\omega_o t + 36.9^o)$

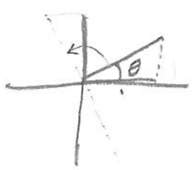
73.8^o e) $f(t) = 10V\cos(\omega_o t - 53.1^o)$

$$10V\cos(\omega_o t + \theta) = 10V\cos(\omega_o t)\cos(\theta) - 10V\sin(\omega_o t)\sin(\theta)$$

$$-6V = 10V\cos(\theta)$$
$$8V = -10V\sin(\theta)$$

$$\frac{\sin(\theta)}{\cos(\theta)} = \frac{-8V}{-6V} = \frac{4}{3} \quad (\text{QUADRANT III})$$

$$\theta = \operatorname{atan}\left(\frac{4}{3}\right) - 180^o = -126.9^o$$



$$\cos(\theta) = \sin(\theta + 90^o)$$
$$\sin(\theta) = \cos(\theta - 90^o)$$

$$f(t) = 10V\cos(\omega_o t - 126.9^o)$$
$$= 10V\sin(\omega_o t - 36.9^o)$$

DUE TO A TYPO (ANSWER (b) WAS SUPPOSED TO BE sin() INSTEAD OF cos()), THERE IS NO CORRECT ANSWER.

IF YOU NOTE THAT, YOU WILL GET 3 PTS.

THE "BEST" ANSWER OF THOSE GIVEN IS EITHER (d) or (e) BECAUSE THE ARE CLOSEST IN TERMS OF PHASE SHIFT.

Question 5. The Fourier coefficients for a triangular wave with amplitude Vm are

$$b_k = \frac{8V_m}{(\pi k)^2}\sin\left(\frac{\pi k}{2}\right)$$

What fraction of the total power is contained in the fundamental component?

a) More than 99%.

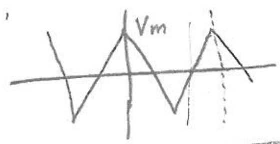
b) More than 95% but less than 99%.

c) More than 90% but less than 95%.

d) More than 75% but less than 90%.

e) 75% or less.

$b_1 = \frac{8V_m}{\pi^2}\sin\left(\frac{\pi}{2}\right) = \frac{8}{\pi^2}V_m$



$P_{AVG} = \frac{4}{T}\int_0^{\frac{T}{4}} V_m^2\left(\frac{t}{T/4}\right)^2 dt$

$= \left(\frac{4}{T}\right)^3 V_m^2 \int_0^{\frac{T}{4}} t^2 dt$

$= \frac{1}{3}\left(\frac{4}{T}\right)^3 V_m^2 \, t^3\Big|_0^{\frac{T}{4}}$

$P_{AVG} = \frac{V_m^2}{3}$

$\frac{\left(\frac{b_1}{\sqrt{2}}\right)^2}{P_{AVG}} = \frac{64 V_m^2 \, 3}{\pi^4 V_m^2 \cdot 2} = \frac{3 \cdot 32}{\pi^4} = 98.6\%$

Question 6. The line spectra for a periodic function exhibits which of the following?

a) Amplitude and phase even.

b) Amplitude even, phase odd.

c) Amplitude odd, phase even.

d) Amplitude off, phase odd.

e) None of the above.

Question 7. Why can we not simply take the Laplace transform of a function and replace s with jω to obtain the Fourier transform of that same function?

a) Because the Fourier integral must be evaluated in order to determine the proper range of ω.

b) Because the Laplace transform does not contain all of the information that the Fourier transform does.

c) Because the Laplace transform involves complex frequencies.

d) Because we need to replace it is –jω instead.

e) We can, provided the function is identically zero for all time less than zero and that the Fourier integral converges.

Question 8. What is the significance of Parseval's Theorem?

a) It guarantees that the conservation of energy is satisfied.

b) It allows us to take the Fourier Transform of signals that might not converge properly.

c) It permits us to determine how much energy in a signal is associated with frequency bands of interest.

d) It allows us to determine what bandwidth is required to minimize harmonic distortion.

e) It allows us to combine multiple functions into a single transform.

Question 9. What is the practical difference between performing circuit analysis using the Laplace versus the Fourier transform?

a) The Fourier transform requires you to determine the initial conditions at t=0s while the Laplace transform does not.

b) The Fourier transform converges for a wider range of functions than the Laplace Transform does.

c) The Laplace transform incorporations initial conditions explicitly while the Fourier transform accomplishes the same thing by allowing driving waveforms to be defined for all time.

d) The Fourier transform only captures the sinusoidal steady state response while the Laplace transform also captures the transient response.

e) The Fourier transform can only be used for signals that are periodic, while the Laplace transform can also deal with aperiodic signals.

Question 10. Multiplication in the time domain equates to what operation in the frequency domain?

a) Multiplication

b) Convolution

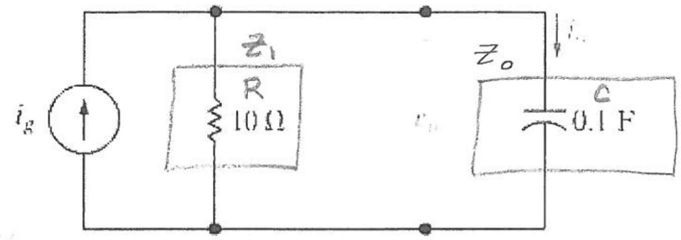
c) Integration

d) Differentiation

e) None of the above.

Problem Section. Show your work! Answers without the reasoning clearly shown, even if correct, will not receive full credit. Systematic failure to track and check units will result in a 40% reduction with an additional 40% reduction if failing to do so results in and error that could/should have been caught. TRACK AND CHECK YOUR UNITS!

Problem 1. (20 points) Given that $i_g(t) = 3A\, e^{\left(-\frac{|t|}{200ms}\right)}$, use the Fourier transform to find $I_o(\omega)$.



$$I_o(\omega) = I_G(\omega)\, H(\omega)$$

$$I_G(\omega) = \mathcal{F}\left\{3A\, e^{\frac{-|t|}{200ms}}\right\}$$

$$= 3A\left(\frac{2a}{a^2+\omega^2}\right);\ a = \frac{1}{200\,ms}$$

$$I_G(\omega) = 30\,\tfrac{A}{S}\left(\frac{1}{\left(\frac{1}{200ms}\right)^2+\omega^2}\right)$$

$$H(\omega) = \frac{Z_1}{Z_o+Z_1} \quad \text{(CURRENT DIVISION)}$$

$$= \frac{R}{R+\frac{1}{j\omega C}}$$

$$= \frac{j\omega RC}{1+j\omega RC}\ ;\ RC = 10\Omega \cdot 0.1F = 1\,sec$$

$$= \frac{j\omega}{\frac{1}{RC}+j\omega}$$

$$H(\omega) = \frac{j\omega}{(1\,\text{r}/s + j\omega)}$$

$$\underline{\underline{I_o = \frac{(30\,A/s)(j\omega)}{\left(\left(\frac{1}{200ms}\right)^2+\omega^2\right)\left(1\,\text{r}/s + j\omega\right)}}} \longleftarrow \text{(Prob \#1)}$$

Problem 2. (20 points) In the following problem, you are given that

$$V_o(\omega) = \frac{2400\frac{V}{s^2}}{j\omega\left(j\omega + 10\frac{r}{s}\right)\left(j\omega + 40\frac{r}{s}\right)}$$

17.26 The voltage source in the circuit in Fig. P17.26 is given by the expression

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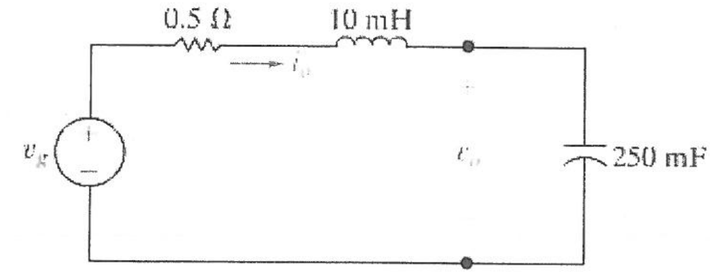
$$v_g = 3\,\mathrm{sgn}(t)\ \mathrm{V}.$$

a) Find $v_o(t)$.

Figure P17.26



$$v_o(t) = \mathcal{F}^{-1}\{V_o(\omega)\}$$

$$= 2400\frac{V}{s^2}\,\mathcal{F}^{-1}\left\{\frac{K_1}{j\omega} + \frac{K_2}{(j\omega + 10\,r/s)} + \frac{K_3}{(j\omega + 40\,r/s)}\right\}$$

$$K_1 = \left.\frac{1}{(j\omega + 10\,r/s)(j\omega + 40\,r/s)}\right|_{j\omega = 0} = \frac{1}{400\,(r/s)^2}$$

$$K_2 = \left.\frac{1}{j\omega(j\omega + 40\,r/s)}\right|_{j\omega = -10\,r/s} = \frac{1}{(-10\,r/s)(-10\,r/s + 40\,r/s)} = \frac{-1}{300\,(r/s)^2}$$

$$K_3 = \left.\frac{1}{j\omega(j\omega + 10\,r/s)}\right|_{j\omega = -40\,r/s} = \frac{1}{(-40\,r/s)(-40\,r/s + 10\,r/s)} = \frac{1}{1200\,(r/s)^2}$$

$$\mathcal{F}^{-1}\left\{\frac{K_1}{j\omega}\right\} = \frac{K_1}{2}\,\mathrm{sgn}(t)$$

$$\mathcal{F}^{-1}\left\{\frac{K_2}{(j\omega + 10\,r/s)}\right\} = K_2\,e^{-(10\,r/s)t}\,u(t)$$

$$\mathcal{F}^{-1}\left\{\frac{K_2}{(j\omega + 40\,r/s)}\right\} = K_3\,e^{-(40\,r/s)t}\,u(t)$$

$$v_o(t) = 2400\frac{V}{s^2}\left[\frac{1}{2\cdot 400\,(r/s)^2}\,\mathrm{sgn}(t) - \frac{u(t)}{300\,(r/s)^2}\,e^{-(10\,r/s)t} + \frac{u(t)}{1200\,(r/s)^2}\,e^{-(40\,r/s)t}\right]$$

$$v_o(t) = 3V\,\mathrm{sgn}(t) - \left[8V\,e^{-(10\,r/s)t} - 2V\,e^{-(40\,r/s)t}\right]u(t) \leftarrow \text{(PROB \#2)}$$