On the Energy Capture of Ultrawide Bandwidth Signals in Dense Multipath Environments

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***Abstract*—A quasi-analytical/experimental analysis is described in this paper to quantify the tradeoff between energy capture and diversity level in a Rake receiver using measured received waveforms obtained from ultrawide bandwidth signal propagation experiments.**

***Index Terms*—Diversity, multipath channel, propagation experiment, Rake receiver, ultrawide bandwidth signal.**

I. Introduction

RECENT WORK has shown that the ultrawide bandwidth (UWB) spread-spectrum radio is a viable candidate for short range multiple-access communications in dense multipath environments because of its fine time resolution properties [1], [2]. To carry out realistic tradeoffs and accurate performance prediction of such systems requires the knowledge of UWB propagation environments. This letter describes a *quasi-analytical/experimental analysis* to quantify the realistic tradeoff between energy capture and Rake diversity level in dense multipath environments. The discussions of classical Rake receivers can be found in [3], [4]. The techniques for generating UWB signals have been around for more than three decades [5]. A comprehensive reference (a total of 82 references) can be found in the excellent tutorial survey paper by Bennett and Ross [6].

The received signal can be modeled as

$$r(u, t) = s(u_s, t) + n(u_n, t) \tag{1}$$

where $n(u_n, t)$ represents the receiver noise as well as the undesired interference. The quantity $s(u_s, t)$ is the channel response to the transmitted UWB signal in the absence of undesired interference and receiver noise. For the case of free-space propagation (absence of multipath), $s(u_s, t) = c(u_s)w(t)$ where $w(t)$ is the ideal received waveform and $c(u_s)$ models the free-space path loss.

Let $k \in \{s, n\}$, the parameter u_k indexes the outcome of a probability space $(\Omega_k, \mathcal{F}_k, \mu_k)$, where Ω_k is a set of outcomes, $\mathcal{F}_k$ is a σ-algebra denoting a set of events, and μ_k: $\mathcal{F}_k \rightarrow [0, 1]$ is a probability measure defined on $(\Omega_k, \mathcal{F}_k)$. The parameter u indexes the outcome of a stochastic environment $(\Omega, \mathcal{F}, \mu)$, defined by a product of $(\Omega_k, \mathcal{F}_k, \mu_k)$ as follows. Let $\Omega = \Omega_s \times \Omega_n \triangleq \{u = (u_s, u_n)\colon u_k \in \Omega_k\}$, $\mathcal{F} = \mathcal{F}_s \times \mathcal{F}_n \triangleq \sigma$-algebra generated by sets $A_s \times A_n$ with $A_k \in \mathcal{F}_k$. It can be shown that there is a *unique* probability measure μ on $\mathcal{F}$ such that $\mu(A_s \times A_n) = \mu_s(A_s) \times \mu_n(A_n)$ [7]. The stochastic environment $(\Omega, \mathcal{F}, \mu)$, in the context of this paper, is a measurement experiment performed in an office building, where $r(u, t)|_{u=\tilde{u}=(\tilde{u}_s, \tilde{u}_n)}$ denotes a *particular* observation, $s(u_s, t)|_{u_s=\tilde{u}_s}$ denotes the channel response to the transmitted UWB signal at a *specific* position $\tilde{u}_s$ inside an office, and $n(u_n, t)|_{u_n=\tilde{u}_n}$ is the *observation* noise.

If the measurement process of $r(u, t)$ is carried out using a digital sampling oscilloscope, then $n(u_n, t)$ can be well modeled as additive white Gaussian noise (AWGN). In this case the detection problem in a free-space environment for the time-shift modulations becomes a coherent detection of equal energy signals in AWGN. Therefore, the optimum receiver is a matched filter or correlation receiver where reference template signals are time shifted versions of $w(t)$ [8]. For a specific observation $r(u, t)|_{u=\tilde{u}}$ in the presence of multipath, the question is how well one can match the signal $s(u_s, t)|_{u_s=\tilde{u}_s}$ with the template consisting of weighted time-shifted versions of $w(t)$. Specifically, the template waveform built into the receiver for use in Rake correlator structures is modeled as

$$\sum_{i=1}^{L_p} c_i w(t - \tau_i). \tag{2}$$

The parameters c_i and τ_i are modeled as *continuous* variables. A typical shape of the waveform $w(t)$ is shown in Fig. 1, and it is assumed that $w(t)$ is reasonably similar to the isolated path signals in the whole ensemble of measured received waveforms.

The c_i's and τ_i's are estimated using a maximum-likelihood (ML) technique for each observation $r(\tilde{u}, t)$. In designing the Rake receiver, it is desirable to quantify the diversity level, L_p, so that the constructed waveform adequately captures the received signal energy. The quantity *energy capture* is defined mathematically for use as a performance measure of the Rake

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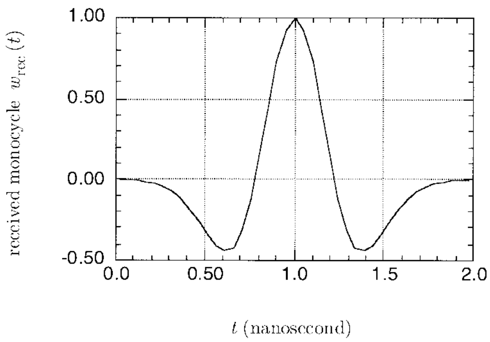


Fig. 1. A typical idealized waveform as a function of time in nanoseconds. The model used in this plot is $w_{\rm rec}(t+1.0) = [1-4\pi(t/\tau_m)^2]\exp[-2\pi(t/\tau_m)^2]$ with $\tau_m = 0.781\,25$, which is assumed to be reasonably matched to the isolated paths in the whole ensemble of measured received waveforms.

receiver. Energy capture as a function of diversity level is evaluated for each of the experimentally measured received waveforms, and typical results are presented.

II. Quasi-Analytical/Experimental Analysis

A. Review of the UWB Signal Propagation Experiments

An UWB signal propagation experiment was performed in a typical modern office building [2]. The bandwidth of the signal used in this experiment was in excess of 1 GHz, resulting in a differential path delay resolution of less than a nanosecond. The transmitter is kept stationary in the central location of the building. Multipath profiles were measured using a digital sampling oscilloscope on one floor at various locations in 14 different rooms and hallways. In each office, multipath measurements were made at 49 different locations. They are arranged spatially in a level 7×7 square grid with 6-in spacing, covering $3' \times 3'$.

Measurements from three different offices are used in the following discussions as typical examples of propagation environments. In these offices, the receiving antennas were located 6, 10, and 17 m away from the transmitter, representing typical UWB signal transmissions characterized as a "high signal-to-noise ratio (SNR)" environment, "low SNR" environment, and "extremely low SNR" environment, respectively. The transmitter and receiving antenna are located in different rooms in these examples. Detailed results of the UWB signal propagation experiment can be found in [2].

B. Rake Receiver's Optimal Template

Given a specific received signal $r(\tilde{u}, t)$ over an observation interval $[0, T)$, and for a given diversity level L_p, the goal is to find the "best" values of $\{\hat{c}_i(\tilde{u})\}_{i=1}^{L_p}$ and $\{\hat{\tau}_i(\tilde{u})\}_{i=1}^{L_p}$, such that the synthesized waveform $\sum_{i=1}^{L_p} \hat{c}_i(\tilde{u}) w(t-\hat{\tau}_i(\tilde{u}))$ is well matched to the received waveform $r(\tilde{u}, t)$.

In this letter, the ML technique is considered as the optimality criterion. For a given L_p, let $\mathbf{c}$ and $\boldsymbol{\tau}$ be the amplitude and the delay vectors, with dimension $L_p \times 1$, defined by

$$\mathbf{c} \triangleq \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_{L_p} \end{bmatrix} \quad \text{and} \quad \boldsymbol{\tau} \triangleq \begin{bmatrix} \tau_1 \\ \tau_2 \\ \vdots \\ \tau_{L_p} \end{bmatrix}. \tag{3}$$

Since $n(u_n, t)$ is modeled as AWGN, the ML criterion is equivalent to a minimum mean squared error criterion. Thus the ML estimates of the amplitude vector $\hat{\mathbf{c}}(\tilde{u})$ and delay vector $\hat{\boldsymbol{\tau}}(\tilde{u})$ based on a specific observation $r(\tilde{u}, t)$ are the values $\mathbf{c}$ and $\boldsymbol{\tau}$ which minimizes the following mean squared error:

$$\mathcal{E}(\tilde{u}, L_p) = \int_0^T \left| r(\tilde{u}, t) - \sum_{i=1}^{L_p} c_i w(t-\tau_i) \right|^2 dt. \tag{4}$$

The minimum value of the above mean squared error is denoted by $\mathcal{E}_{\min}(\tilde{u}, L_p)$.

For a wide bandwidth transmission channel, it is often assumed that the propagation channel is separable, i.e.,

$$|\tau_i - \tau_j| > D, \qquad \text{for all } i \neq j \tag{5}$$

where $D \triangleq$ the width of $w(t)$. In this case,

$$\int_0^T w(t-\tau_i) w(t-\tau_j)\, dt = 0, \qquad \text{for all } i \neq j. \tag{6}$$

Under the assumption of a separable multipath channel, the ML estimates of $\hat{\boldsymbol{\tau}}(\tilde{u})$ and $\hat{\mathbf{c}}(\tilde{u})$ based on $r(\tilde{u}, t)$ were derived in [9], and it was shown to result in decoupled solutions as

$$\hat{\boldsymbol{\tau}}(\tilde{u}) = \underset{\boldsymbol{\tau}}{\operatorname{argmax}} \left\{ \sum_{i=1}^{L_p} \left| \frac{\chi(\tilde{u}, \tau_i)}{\sqrt{R(0)}} \right| \right\} \tag{7}$$

$$\hat{\mathbf{c}}(\tilde{u}) = \frac{\chi(\tilde{u}, \hat{\boldsymbol{\tau}}(\tilde{u}))}{R(0)} \tag{8}$$

where $R(0) = \int_0^T |w(t)|^2\, dt$, and the elements of $\boldsymbol{\tau}$ satisfy the condition given in (5). The $\chi(\tilde{u}, \tau_i)$ is the ith component of the vector $\chi(\tilde{u}, \boldsymbol{\tau})$ which is given by

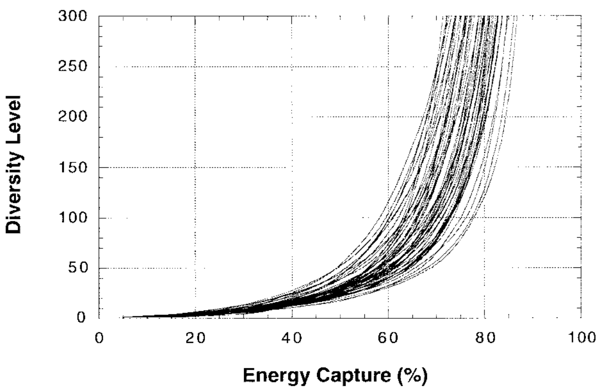
$$\chi(\tilde{u}, \boldsymbol{\tau}) = \int_0^T r(\tilde{u}, t) \begin{bmatrix} w(t-\tau_1) \\ w(t-\tau_2) \\ \vdots \\ w(t-\tau_{L_p}) \end{bmatrix} dt. \tag{9}$$

The above integral can be interpreted as the correlation of the received signal $r(\tilde{u}, t)$ with $w(t)$ at different hypothesized delays.

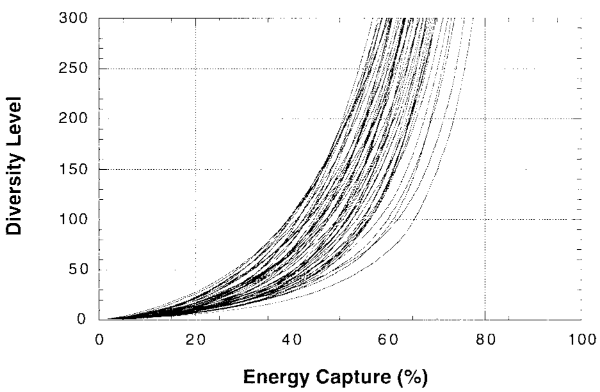
III. Performance Measure and Results

The performance of a Rake receiver can be measured in terms of the quantity *energy capture*. Energy capture as a function of L_p for each observation $r(\tilde{u}, t)$ is defined mathematically as

$$\mathrm{EC}(\tilde{u}, L_p) = 1 - \underbrace{\frac{\mathcal{E}_{\min}(\tilde{u}, L_p)}{E_{\rm tot}(\tilde{u})}}_{\triangleq\ \text{normalized MSE}} \tag{10}$$



(a)



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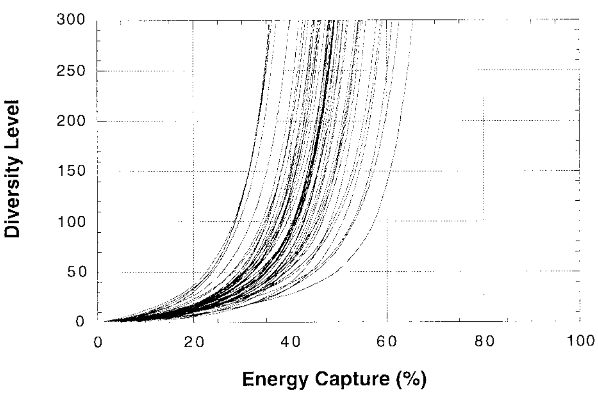

(c)

Fig. 2. The required diversity level, L_p, in a UWB Rake receiver as a function of percentage energy capture for each of the 49 received waveforms in an office representing: (a) a "high SNR" environment; (b) a "low SNR" environment; and (c) an "extremely low SNR" environment.

where $E_{\mathrm{tot}}(\tilde{u})$ is the total energy of the received waveform $r(\tilde{u}, t)$ given by $E_{tot}(\tilde{u}) = \int_0^T |r(\tilde{u}, t)|^2 \, dt$. This can be interpreted as the portion of the received waveform energy captured by the Rake receiver with diversity level L_p.

Energy capture as a function of the diversity level, L_p, is computed for each received waveform measurement. The required diversity level in a UWB Rake receiver versus percentage energy capture is plotted in Fig. 2 for each of the 49 received waveforms measured in three different typical environments.[1] Note that the amount of energy capture increases rapidly as the diversity level increases from 0 to 50. However, this improvement becomes gradual and only a negligible improvement can be made as the diversity level increases beyond 100. In practice, UWB Rake receivers are designed to operate in regions where the increase in energy capture as a function of diversity level is rapid.

IV. Conclusions and Caveats

Energy capture as a function of diversity level used in a UWB Rake receiver is evaluated for each of the experimentally measured received waveforms. The results show that a diversity level less than 50 is sufficient for an UWB Rake receiver in a typical modern office building.

The results presented in this paper are based on the simple template structure given by (1). However, more complicated template structures are possible at the expense of receiver complexity. One such example is the use of a family of basic waveforms $\{w^l(t)\}_{l=1}^N$ and determination of the "best" choice of $\{\{\hat{c}_i^l(\tilde{u})\}_{i=1}^{L_p}, \{\hat{\tau}_i^l(\tilde{u})\}_{i=1}^{L_p}\}_{l=1}^N$ such that the synthesized waveform $\sum_{l=1}^N \sum_{i=1}^{L_p} \hat{c}_i^l(\tilde{u}) w^l(t - \hat{\tau}_i^l(\tilde{u}))$ is well matched to the received waveform $r(\tilde{u}, t)$.

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[1] The "usual" separable channel assumptions given in (5) may not always be valid, even in the case of UWB transmissions channels. However the ML estimators derived under this assumption given in (7) and (8) are used in computations for simplicity.