Signature Coding over Multiple Access OR Channel

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Motivated by information retrieval systems, Kautz and Singleton [1] introduced the concept of superimposed codes. Much later this code property had an other interpretation. If m active users out of T potential users communicate with such a code via a multiple access OR channel, one can detect the active set of users.

In this paper superimposed codes are used for signature coding. Signature coding means that there are T users in the communication system. All the users have their own code word. If a user is active then it sends his code word into the channel, and otherwise does nothing, formally sends the all-zero code word into the channel. The decoder, from the Boolean sum of the code words of the active users, should reconstruct the set of active users.

Our goal is to minimize the code length for given m and T. The minimum code length of superimposed code is denoted by $N(T, m)$

There are some bounds on the minimal code length $N(T, m)$, based on the work of Dyachkov, Rykov [2] and A, Zeisel [3].

A superimposed code construction usually assumes that at most m user communicate on the multiple access OR channel at the same time, where m is a fixed number. Decoding error occurs if more than m users communicate simultaneously. For a given $\varepsilon > 0$ we want to have the decoding error probability smaller than ε.

Let us introduce a new model for the activity of users. Assume that there are T potential users, each is active with probability q in a time-slot. Then the number M of simultaneously active users is a binomial random variable with parameters (T, q).

We can use a not superimposed code construction that does not guarantee error-free decoding if the number of active users is below the limit m, but ensure that the sum of at most m code word includes a foreign code word with small probability. In this case the probability of erroneous decoding consists of two terms. Decoding fails if more than m users communicate simultaneously or the sum of code words of active users' cover a code word which is not in this set.

$$\mathbf{P}\left(\text{decoding error}\right) \leq \mathbf{P}\left(M > m\right) + \mathbf{P}\left(\text{false covering}\right).$$

Kautz and Singleton [1] suggested a code construction with length N_{KS} which eliminates the false covering error by using a special code concatenation method.

In random constant weight code construction method we generate binary code words of length N_r which consist of L-long segments, so all code word contains $\frac{N_r}{L}$ segments. Each segment has exactly one bit 1 such that the location of this 1 is uniformly distributed. The optimal segment-length is $L := \frac{M}{\ln 2}$.

Code length N_{KS} or N_r is chosen so, that $\mathbf{P}\left(\text{decoding error}\right)$ be smaller than ε. Comparing the Kautz–Singleton and random code constructions in numerical examples, the latter prove to be superior, and the resulted code length is pretty much below the existing upper bounds for error-free decoding.

A measuring example is considered in which we want to collect power consumption data of customers automatically. The power line can be used as a multiple access OR channel [4]. The measuring instrument of a user sends its unique code word to this common channel if a user has consumed a unit (e.g. 1 kWh) of electric energy, so signature coding is needed. If the channel is synchronized to the frequency 50 Hz of the power line, and the time-slot is one sinus period (20 ms), the parameter q of the binomially distributed random variable for the number of active users is around 10^{-5}. If we would like to guarantee that decoding error probability is below 10^{-5} then we need channel speed 11 kbps.

References

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