On Multiple Access Interference in a DS/FFH Spread Spectrum Communication system

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Abstract
This paper describes the influence of Multiple Access (MA) interference on the performance of a non-coherent hybrid Direct Sequence Fast Frequency Hopping SSMA Communication system applying FSK-modulation. The MA-interference plays an important role in determining the total interference in a Spread Spectrum system, especially in a non-cellular environment, where Power Control is hardly possible. A cross correlation factor which is directly responsible for this interference is derived. Using this factor, a comparison between slow and fast frequency hopping (both in combination with direct sequence) is made. At the end two sets of Kasami codes are selected which offer a good performance.

1 Introduction
This paper deals with a hybrid Direct Sequence, Fast Frequency Hop system in a non-cellular environment. Whereas most publications so far (1, 2 and others) put emphasis on long hopping-patterns, we focus on short sequences resulting in less bandwidth occupancy. We also assume a spatial distribution of users which results in non-equal power reception for different users.

A non-cellular environment leads to a point-to-point communication system without a base station. This is a more flexible and less expensive method than the cellular approach, but inhibits power control, playing a key role in reducing the Near-Far effect [3, 4]. The hybrid DS/FFH technique is applied to both beat the Near-Far effect and to retain the advantages of Direct Sequence: Jamming rejection, fading rejection and Security.

In section 2 the hybrid system-model is described. Section 3 deals with the MA-interference, here the measure for MA-interference in a DS/FFH system is derived, and the relation with the S/I-ratio is mentioned. In section 4 a comparison is made between DS/SH and DS/FFH for Rayleigh fading channels. Section 5 provides two sets of Kasami codes which offer good MA-interference properties. Finally section 6 gives some conclusions.

2 System Model
2.1 DS/FFH Spread Spectrum technique
The DS/FFH Spread Spectrum technique combines the advantages of the direct-sequence and frequency-hopping spreading techniques while compensating for the others. Each data bit is divided over a number \(N_{FH}\) of frequency-hop channels (carrier frequencies). In each frequency-hop channel a complete PN-sequence of length \(N\) is combined with the data signal (see figure 1 where \(N_{FH} = 5\)). Applying Fast Frequency Hopping (FFH) requires a wider bandwidth than Slow Frequency Hopping (SFH). This increase however, is marginal compared to the enormous bandwidth already in use.

![Figure 1: DS/FFH Spreading scheme](image)

Since the FH-sequence and the PN-codes are coupled, every receiver is identified by a combination of an FH-sequence and \(N_{FH}\) PN-codes. To limit the probability that two users share the same frequency channel simultaneously, frequency-hop sequences are chosen in such a way that two transmitters with different FH-sequences share at most two frequencies at the same time in one bit-period (asynchronous transmission).

Applying several frequency hops within each databit disqualifies modulation by some kind of Phase Shift Keying. PSK is also quite susceptible to channel distortions. An FSK modulation scheme is therefore chosen.

2.2 The DS/FFH SSMA system model
One of the important advantages of spread spectrum systems is their multiple access capability. The interference that results from simultaneous transmission is called 'Multiple Access (MA) Interference'. Contribution to this interference occurs when the reference user and a non-reference user use the same FH-channel for a fraction of a frequency-hop.

In case of pure DS all codes are transmitted in the same frequency slot, so the codes will correlate completely. In the DS/FFH case, two subsequent codes are transmitted in different frequency slots, so those codes do correlate only partially. In figure 2 this is shown in more detail.

The DS/FFH SSMA system model that will be considered is shown in figure 3.

Suppose there are \(K\) active users, one of them, user \(i\), being the reference user. The other \(K - 1\) users cause MA-interference.
with an output

\[ Z_t = \sqrt{E_P} \sum_{k=1}^{K} R_{k,i}(\tau_k) \cos(\phi_k) \]
\[ + \int_0^{T_R} n(t) a(t) \cos(\cdot) dt \]

Here \( \phi_k = \theta_k + \omega_k \tau_k \), \( \tau_k \) is the delay of the \( k \)-th user (\( \tau_k \) is defined to be positive if user \( i \) is delayed with regard to user \( k \)). So \( \tau_k \) can vary between \(-T_h\) and \( T_h\).

\( n(t) \) is the channel noise, \( \cos(\cdot) \) denotes the same cos-term as in (2) and \( R_{k,i}(\tau_k) \) is a continuous time partial cross correlation function defined by:

\[ R_{k,i}(\tau_k) = \int_{-T_R/2}^{T_R/2} a_k(t + \tau_k) a_i(t) dt \]

In the DS-case there are two correlation terms [5]. The interval \( \tau_{12}, \tau_{23} \) is the time that two users \( i \) and \( k \) share the same frequency-hop channel. \( \tau_{12} \) and \( \tau_{23} \) are related to \( \tau_1 \) in the following way (see also figure 2):

\( \tau_1 = \begin{cases} 0, & \tau_1 \leq 0 \\ \tau_1, & \tau_1 > 0 \end{cases} \)

\( \tau_2 = \begin{cases} \tau_1 + T_h, & \tau_1 \leq 0 \\ \tau_1, & \tau_1 > 0 \end{cases} \)

If \(-T_h \leq \tau_1 \leq (i + 1)T_h \leq T_h\), \( J \) being an integer, this correlation function can be written as:

\[ R_{k,i}(\tau_k) = C_{i,j,l}(i + 1)(\tau_1 - \tau_k) + C_{k,i,l}(i + 1)(\tau_1 - \tau_k) \]

The data signal, \( b_i(t) \), is a polar bit stream of unit amplitude, with a duration \( T_b \). This signal is FSK-modulated. The FSK-spacing is \( \Delta f_{FSK} \). \( a_i(t) \) is the waveform of the PN-code, also a sequence of polar bits with unit amplitude, with duration \( T \). If we denote by \( a_i(t) \) the sequence \([-\Delta f_{FSK} + \frac{T}{2}, -\Delta f_{FSK} + \frac{T}{2}, \ldots, \Delta f_{FSK} - \frac{T}{2}, \Delta f_{FSK} - \frac{T}{2}, \ldots] \) of duration \( T_h = N T_F = T_h / T_F \), the transmitted signal of the \( k \)-th user is:

\[ s_k(t) = \sqrt{E_P} a_k(t) + b_i(t) \delta(t) \]
As can take values between $-T_i$ and $7$, $T_i$ is integrated over the corresponding interval. Substituting $R_{i+}(T)$ from (5) and evaluating the integral gives:

$$\text{Var}(Z_{i,MA}) = \frac{P_{th}}{8T_i} \sum_{k=1}^{K} \int_{-T_i}^{T_i} R_{i+}^2(\tau) d\tau$$

$$= \frac{P_{th}}{8T_i} \sum_{k=1}^{K} \sum_{j=-N}^{N} \int_{-T_i}^{T_i} R_{i+}^2(\tau) d\tau + \frac{N_0 T_i}{4}$$

As $\tau$ can take values between $-T_i$ and $T_i$, $\tau$ is integrated over the corresponding interval. Substituting $R_{i+}(T)$ from (5) and evaluating the integral gives:

$$\text{Var}(Z_{i,MA}) = \frac{P_{th}}{24N_i^3} \left( \sum_{k=1}^{K} p_{k,i} \right) + \frac{N_0 T_i}{4}$$

where:

$$p_{k,i} = \sum_{j=-N}^{N} \left[ C_{i,j}^2(1) + C_{i,j}(1)C_{i,j}(1 + 1) + C_{i,j}^2(1 + 1) \right]$$

After some calculation we find for the mean and variance of $p_{k,i}$:

$$E[p_{k,i}] = 2N_i - 1$$

$$\text{Var}(p_{k,i}) = 6N_i^2 - 5N_i^2 + 2N_i$$

Interesting to mention is the relation between this factor and the $r_{k,i}$-factor from (6); $r_{k,i} = r_{k,i} - 1$. Since the $p_{k,i}$-term is directly responsible for the MA-interference in a DS/FFH system, proper PN-codes can be selected on the basis of this term.

Assuming the mean value for $p_{k,i}$, the resulting signal to interference ratio ($S/I$) becomes:

$$S/I_{\text{FFH}} = \frac{2E_{k,i}/N_0}{1 + \frac{1}{6N_i} \sum_{k=1}^{K} E_{k,i}}$$

Here $E_{k,i}$ is the energy per frequency hop.

One last remark is to be made. In the derivation of the signal to interference ratio the signals are assumed to be Gaussian, this assumption is only valid for either long PN-codes or a large number of users. Simulations show that for a FH-pattern with maximal two hits and a PN-code length of 63, 2 interfering users (equal power) justify this assumption.

$$S/I_{\text{SFH}} = \frac{2E_{k,i}/N_0}{1 + \frac{1}{3N_i} \sum_{k=1}^{K} E_{k,i}}$$

Here $E_{k,i}$ is the energy per frequency hop.

4 FFH versus SFH

This section provides a comparison of the performance of DS/FFH and DS/SFH systems, on the basis of the same bandwidth occupancy. As this section is only meant as an illustration, we will not go into detail but give references instead.

For $p_{k,i}$ a value of $2N_i^2$ is chosen ($E[p_{k,i}] = 2N_i^2 - 1$), in section 5 will be shown that sets of PN-codes can be found which have $2N_i^2$ as an upper bound. For $N_{\text{FFH}}$ we chose a value of 7. Now there are 6 sequences possible with a length of 7 which share maximally two frequencies simultaneously during 1 hop-time (random time shift). For the DS-spread system the PN-code length is 63.

The chance that two transmitters share the same frequency slot simultaneously ($P_{\text{hit}}$) is evaluated in [1] and is equal to $1/N$, with $N$ being the number of frequency-hop channels. For the SFH case we find the following formula:

$$P_{\text{hit}, \text{SFH}} = \frac{1 - N_i^{-1}}{q} + \frac{2}{N_i^2}$$

with $N_i$ equal to the number of bits per hop.

For the $S/I$-ratio the formula by Pursley will be used [6] (The $r_{k,i}$-factor will also be approximated by its mean: $2N_i^2$):

$$S/I_{\text{SFH}} = \frac{2E_{k,i}/N_0}{1 + \frac{1}{3N_i} \sum_{k=1}^{K} E_{k,i}}$$

As no power control is assumed an assumption towards the spatial distribution of the users is to be made. For this we will use the log-normal distribution function motivated in [9]:

$$K'(r) = \frac{K}{2\pi r_0} \exp \left( - \frac{(\ln r)^2}{2r_0^2} \right)$$

Here $K'(r)$ is the number of interfering users per unit distance at a distance $r$. $r_0$ is a parameter which determines the shape of the distribution, this parameter is chosen to have a value of 0.7.

The power is expected to decrease as a function of the normalized distance: $P(r) \sim r^{-\beta}$, for $\beta$ a typical value of 3 has been chosen.

The reference user is assumed to have a normalized distance

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When comparing DS/FFH and DS/SFH without applying the diversity inherent to FFH both systems will perform about the same. There is only an advantage of using FFH over SFH if \( N_{k} \) the number of bits per frequency hop, is limited. A limitation on this point can be introduced by an applied error correcting code (and so by the amount of memory in the receiver). In figure 5 the BER versus \( E_b/N_0 \) curves are drawn for SFH \( (N_{k} = 50) \) and FFH, in this example \( \gamma \) and \( N_{FFH} \) have a value of 7. The number of active users \( (N_{k}) \) is varied between 1 and 51. Figure 6 shows the influence of changing the number of bits per hop for DS/SFH \( (N_{k} = 31, \gamma = N_{k} = 7) \).

More interesting is the case in which we apply diversity (majority voting). Results are given in figure 7 for 1, 11, 31 and 51 users, \( \gamma \) and \( N_{SFH} \) have a value of 7. For majority voting it is assumed that the interference of two successive hops is independent (different frequency channels, pseudo-random hop-sequences), this is not true in the DS/SFH case.

5 Finding a proper set of PN-codes

In section 3 a measure was derived for the MA-interference in a DS/FFH SSC system. This measure will function as a criterion to select a proper set of PN-codes. Except for a low cross-correlation factor the codes also have to meet the "balance"-property: the difference between the number of ones and zeros in the code may only be 1. This last requirement stands for good spectral density properties. Candidates are codes from the large set of Kasami codes, the code-length is chosen to be 63. In [11] also code-set selections can be found, candidates here are the Gold codes and the small set of Kasami codes, the "balance"-criterion is not met in that case. Kasami codes in the large set are created by combination of 3 m-sequences [8, 12]:

\[
\begin{align*}
\rho & = u \cdot T^{v} \cdot T^{u} w
\end{align*}
\]

Here \( u \) and \( v \) are m-sequences of length: \( N = 2^{\alpha} - 1 \) (n even) which form a preferred pair [13]. \( u \) is a m-sequence resulting after decimation the \( r \)-code with a value \( 2^{\alpha/2} + 1 \). \( T \) denotes a delay of one element, \( k \) is the offset of the \( u \)-code with respect to the \( u \)-code and \( w \) is the offset of the \( r \)-code with respect to the \( r \)-code. Offsets are relative to the all-ones state.

The notation from (16) will be used for Kasami-codes of the large set with \( n = 6 \), \( N = 63, u \) is the m-sequence with feedbacks (6,1) and \( v \) is the m-sequence with feedbacks (6,2,1).

To calculate the total MA-interference in a SSC system, we have to know the \( \rho_{uv} \)-factor of each code combination used. The mean-value of this term is \( 2N^2 - 1 \), as a selection-criterion we require that for all code combinations in the set the \( \rho_{uv} \) -factor has a value less than this mean \( (\rho_{uv} < 2N^2 - 1) \)
The PN-codes in this example have length 63, so there are only 2^{241} possible PN-codes. Let us look at a set of PN-codes, for which all \( p_i \) factors have values below \( 2^{2/3} \). These sets are shown in the tables 1 and 2.

Table 1: set of PN-codes with \( p_{ki} < 2^{3/2} - 1 \) with \( N = 63 \)

<table>
<thead>
<tr>
<th>( k_{24} )</th>
<th>( k_{25} )</th>
<th>( k_{26} )</th>
<th>( k_{27} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>64, 63, 62</td>
<td>50, 51</td>
<td>57, 58</td>
<td>59, 60</td>
</tr>
<tr>
<td>60, 61</td>
<td>64, 65, 66</td>
<td>67, 68, 69</td>
<td>70, 71</td>
</tr>
<tr>
<td>72, 73</td>
<td>74, 75, 76</td>
<td>77, 78, 79</td>
<td>80, 81</td>
</tr>
<tr>
<td>82, 83</td>
<td>84, 85, 86</td>
<td>87, 88, 89</td>
<td>90, 91</td>
</tr>
<tr>
<td>92, 93</td>
<td>94, 95, 96</td>
<td>97, 98, 99</td>
<td>100, 101</td>
</tr>
</tbody>
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Table 2: set of PN-codes with \( p_{ki} < 2^{3/2} - 1 \) with \( N = 63 \)

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</table>

To determine the number of addresses which can be formed with these sets of codes, recall that if there are \((N_{FH} + 1)\) FH-channels available, \( N_{FH} \) FH-sequences can be formed. Another constraint is that one user address uses \( N_{FH} \) different PN-codes (to avoid the case that two addresses have the frequency-hop sequence as well as more than one PN-code in common). When \( N_{FH} = 17 \) (number of codes in a code set) the resulting number of addresses is then about \( 4 \times 10^6 \).

6 Conclusions

To be able to reduce the MA-interference in a DS/FFH SSMA communication system a criterion for choosing sets of PN-codes was provided. First a factor \( p_{ki} \) is derived which is directly responsible for the MA-interference in DS/FFH SSMA systems. Comparing this factor with a corresponding factor in a DS/FH or DS SSC system [6] it appears that, if no diversity is applied, DS/FFH systems only outperform DS/FH if the number of bits per hop in the latter case is limited. However when exploiting the diversity inherent to Fast Frequency Hopping, the DS/FFH system performs much better. This is illustrated by BER versus \( E_b/N_0 \) plots for both techniques.

In most applications a lot of users have to be allowed in the system. In such a case the Bit Error Rate is almost totally determined by the MA-interference. This makes selecting sets of PN-codes with good cross-correlation properties necessary. The sets presented in section 3 are the proper sets to use under these circumstances.

References


