Constant Weight Codes For Multiaccess Channels Without Feedback

Shakir Abdul-Jabbar and Peter de Laval
Department of Electrical Engineering
Linköping University, Sweden

Abstract

The design of superimposed codes for the multiaccess OR-channel is considered. The performance of constant weight (CW) codes when used as superimposed codes is investigated. Several constructions for CW codes are compared : affine geometry codes, projective geometry codes, and codes obtained by code concatenation. A comparison to the sphere packing bound and the Johnson bounds is made.

I. Introduction

Consider the situation when a large number of users share a common channel. The classical solution of fixed assignment (i.e. time division multiple access ,TDMA, or frequency division multiple access ,FDMA) is adequate if most of the users are active most of the time. But if only a small subset is active at any time interval, the fixed assignment solution is clearly inefficient. Superimposed codes can be used in such situations. These codes are especially useful when immediate feedback is not possible, as in satellite channels. Ground stations can ,for example, use these codes to make reservations for data channels. We investigate the performance of a class of codes that can easily be characterized as superimposed codes. This class is CW codes. In section II the system model and formal definitions of the codes are presented. The relation between CW codes and superimposed codes is described in section III. Bounds on superimposed codes and CW codes are given in section IV. In section V several constructions for CW codes are presented and their performance as superimposed codes is analysed.

II. The system model

Before we describe the system model we need some definitions.

Definition *Superposition of binary sequences*

The *superposition* $\mathbf{x} \vee \mathbf{y}$ of two binary {0,1} sequences $\mathbf{x}$ and $\mathbf{y}$ of length n is defined as

$$\mathbf{x} \vee \mathbf{y} \triangleq \mathbf{z} = (z_1, z_2, \dots, z_n)$$

where

$$z_i \triangleq \begin{cases} 0 & \text{if } x_i = y_i = 0 \\ 1 & \text{otherwise} \end{cases}$$

The superposition of a set $A = \{ \mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \dots, \mathbf{x}^{(m)} \}$ of n-dimensional binary sequences is denoted by

$$f(A) \triangleq \mathbf{x}^{(1)} \vee \mathbf{x}^{(2)} \vee \dots \vee \mathbf{x}^{(m)}.$$

Definition *Multiaccess OR-channel*

With a *Multiaccess OR-Channel* we mean a channel that operates on a set A of binary sequences and produces an output sequence $\mathbf{z}$ equal to the superposition of the input set, i.e.

$$\mathbf{z} \triangleq f(A).$$

The correlation between two binary {0,1} sequences $\mathbf{x}$ and $\mathbf{y}$ (abbreviated $c(\mathbf{x},\mathbf{y})$ is simply the number of positions where both have ones.

Definition *Disjunctive Code*

The binary code C with codeword length n and size T is a *disjunctive code* (also called *zero false dropping code*) of order m if each subset $A \subseteq C$ of size $|A| \leq m$ has the property that for every word $\mathbf{x} \in A$ we have $c(\mathbf{x}, f(A)) = w_H(\mathbf{x})$ but for all other words $\tilde{\mathbf{x}} \in C \setminus A$ we have $c(\tilde{\mathbf{x}}, f(A)) < w_H(\tilde{\mathbf{x}})$. The set of all disjunctive codes with parameters n, m and T is denoted $\mathcal{D}($ n,m,T).

The class of *disjunctive codes* is a subset of the class of *superimposed codes* , and were introduced by W.H. Kautz and R.C. Singleton [1]. See also [2].

Definition *Protocol Sequence*

The binary code C with length n and size T is a protocol sequence of order m if any set $A \subseteq C$ of size m or less has the property that any $x \in A$ has at least one position where it has a one where all other codewords in A has a zero (we say x has a free slot). The set of all protocol sequences with parameters n, m and T is denoted by $\mathcal{P}$(n,m,T).

88CH2607-0 C 1988 IEEE

Definition *Constant Weight Code*

The binary code C with codeword length n and size T is a *constant weight code* if all codewords $\mathbf{x} \in C$ have the same Hamming weight $w_H(\mathbf{x}) = w$. One interesting parameter for the constant weight codes is the *maximum correlation* c which is related to the minimum distance d by the identity

$$d = 2 \cdot w - 2 \cdot c \ .$$

The set of all constant weight codes with parameters n, w, c and T is denoted $\mathcal{CW}($ n,w,c,T).

Our system consists of a set of T users that share a multiaccess OR-channel (see fig. 1).We distribute codewords from a superimposed code to all users. We assume block and bit synchronization between the users. The users transmit through the channel. All nonactive users can be thought of as transmitting the all-zero sequence. If the number of active users is less than or equal to m we know from the definition of superimposed codes that we can decompose the received word into its component codewords.If each user is given a set of codewords , information can be communicated. But if every user is given only one codeword we can identify the active users. In this paper we consider only the identification problem. The codeword length (n) in a superimposed code is proportional to the delay. Our objective is to find superimposed codes that have a low n for a fixed m and T.

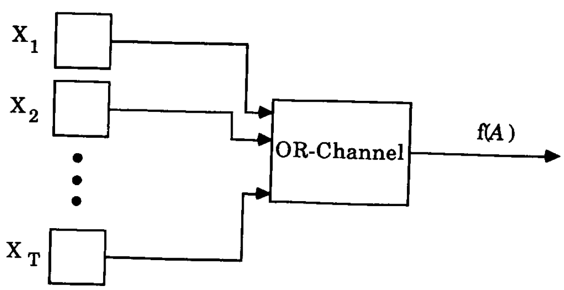

Figure 1. System model

III. Relation between CW codes and superimposed codes

It has been shown (see [1], [2], and [3]) that the following relations are valid.

$$\mathcal{D}(n,m\text{-}1,T) = \mathcal{F}(n,m,T) \qquad (1)$$

$$\mathcal{CW}(n,w,c,T) \subseteq \mathcal{F}(n, \lceil w / c \rceil ,T) \qquad (2)$$

where $\lceil x \rceil$ denotes the lowest integer greater than or equal to x.

Combining (1) and (2) we get

$$\mathcal{CW}(n,w,c,T) \subseteq \mathcal{D}(n, \lceil w/c \rceil -1,T) \qquad (3)$$

Thus any CW code with parameters n,w,c, and T is a superimposed code of length n , size T and order greater than or equal to $\lceil w/c \rceil$ -1. We call $\lceil w/c \rceil$ -1 the designed order and denote it by m_d .

IV. Bounds

(a) A sphere packing bound

Define

$$N_D(m,T) \triangleq \min \{ n: \mathcal{D}(n,m,T) \neq \emptyset \}$$

It has been shown (see [3]) that

$$N_D(m,T) \geq \log_2 \sum_{i=0}^{m} \binom{T}{i}$$

(b) Johnson bounds : The two Johnson bounds for CW codes are upper bounds on the codesize T for fixed n,m and c (see ref. [9] and [10]). We use them in a slightly weaker form.

1) $$T(n,w,c) \leq \frac{\binom{n}{c+1}}{\binom{w}{c+1}}$$

2) $$T(n,w,c) \leq \left\lfloor n \cdot \frac{w-c}{w^2-nc} \right\rfloor \quad \text{iff} \quad w^2 > nc$$

where $\lfloor x \rfloor$ denotes the integer part of x.

These bounds are used indirectly to find the minimum length n of CW codes that are disjunctive codes with size T_o and order m_o. More precisely, we look for sets of parameters n,w, and c satisfying:

1) $T(n,w,c,) \geq T_o$

2) $\lceil w/c \rceil -1 \geq m_o$

Among these sets we find the one with the lowest n.

V. CW codes and their performance as supeimposed codes

Five different families of CW codes are compared. The first two constructions are based on finite geometries. By restricting ourselves to finite fields we obtain the following codes : (see ref. [5] appendix B and ref. [4]) :

(1) Affine geometry codes (AG(k,q)):

$$AG(k,q) \Rightarrow \mathcal{CW}\left(n = q^k, w = q, c = 1, T = \frac{q^k (q^k - 1)}{q (q - 1)} \right)$$

where q is a power of prime.

(2) Projective geometry codes (PG(k,q)):

PG(k,q) ⇒

$$CW\left(n = \frac{q^{k+1}-1}{q-1},\ w = q+1,\ c = 1,\ T = \frac{(q^{k+1}-1)(q^{k}-1)}{(q^{2}-1)(q-1)} \right)$$

where q is a power of prime.

Both constructions are optimum in the sense that they satisfy the first Johnson bound with equality.

The rest of the codes are based on code concatenation.

(3) Concatenated codes :

A concatenated code (see ref. [8]) consists of an outer code and an inner code. The alphabet of the outer code is mapped into codewords from the inner code. We use a Reed-Solomon (RS) outer code and a CW code as inner code. Clearly the resulting code is also a CW code. For the inner code we use : AG(k,p), PG(k,p), and the orthogonal weight one code.
These constructions will be abbreviated by AG, PG, RS/AG, RS/PG, RS/orth.The last three codes are the concatenated codes.

Based on equation (3) we analysed the performance of these codes when used as superimposed codes. Our task now is to find the codes that give the shortest length (i.e. minimum delay) for a fixed m_d and T.

After extensive search it was found that the code RS/orth. gives the minimum delay. Table 1 shows the codeword length for code size 10^4 and 10^7 . For different values of T the pattern is the same.

m_d	AG	PG	RS/AG	RS/PG	RS/Orth
2	625	255	135	130	77
3	625	364	325	341	110
4	625	781	726	651	169
5	1331	781	1089	1064	253
6	1331	1464	1573	1596	299
7	1331	1464	2178	2128	345

(a) $T \geq 10^4$

m_d	AG	PG	RS/AG	RS/PG	RS/Orth
2	15625	8191	275	255	169
3	15625	19531	625	651	256
4	15625	19531	1408	1460	459
5	32768	19531	2304	2263	567
6	32768	37449	3969	3577	675
7	32768	37449	5625	5784	899

(b) $T \geq 10^7$

Table 1. Codeword length n for different classes of codes

Figure 2 shows the length of RS/orth. for $T \geq 10^7$ and m_d from 2 to 20. The bounds in the figure should be interpreted with care. The Johnson bounds give a lower bound for the codeword length n of CW codes under the restriction that $T \geq 10^7$, while the sphere packing bound is a lower bound for n of disjunctive codes under the same restriction. For the Johnson bounds the parameters w and c of the CW code were translated into the parameter m_d of the corresponding disjunctive code (see section III). Table 2 gives the parameters of the concatenated RS/orth. code in detail. The subscripts i and o denote inner and outer code respectively.

It should be noted that an efficient decoding algorithm for this code has been developed (see ref. [11]).

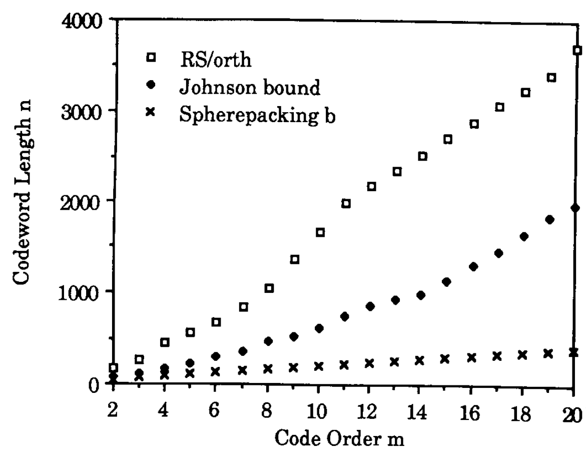

Figure 2. Performance of RS/orth.

$n_o = w$	k_o	$q = n_i$	n	$c=k_o-1$	T≈	m_d
13	7	13	169	6	$6.3\cdot10^7$	2
16	6	16	256	5	$1.7\cdot10^7$	3
17	5	27	459	4	$1.4\cdot10^7$	4
21	5	27	567	4	$1.4\cdot10^7$	5
25	5	27	675	4	$1.4\cdot10^7$	6
29	5	29	841	4	$2.1\cdot10^7$	7
33	5	32	1056	4	$3.4\cdot10^7$	8
37	5	37	1369	4	$6.9\cdot10^7$	9
41	5	41	1681	4	$1.2\cdot10^8$	10
34	4	59	2006	3	$1.2\cdot10^7$	11
37	4	59	2183	3	$1.2\cdot10^7$	12
40	4	59	2360	3	$1.2\cdot10^7$	13
43	4	59	2537	3	$1.2\cdot10^7$	14
46	4	59	2714	3	$1.2\cdot10^7$	15
49	4	59	2891	3	$1.2\cdot10^7$	16
52	4	59	3068	3	$1.2\cdot10^7$	17
55	4	59	3245	3	$1.2\cdot10^7$	18
58	4	59	3422	3	$1.2\cdot10^7$	19
61	4	61	3721	3	$1.4\cdot10^7$	20

Table 2. Parameters for the RS/orth. code

VI. Conclusion

Superimposed codes can be used for unique identification of users sharing a multiaccess OR-channel. Superimposed codes can easily be derived from constant weight codes. A comparison between several families of constant weight codes was made. It was found that using a concatenated code with a Reed-Solomon outer code and an orthogonal weight one inner code gives the lowest block length (i.e. lowest delay).

Acknowledgment

The authors would like to thank professor Thomas Ericson for suggesting the problem and for valuable comments during the preparation of the paper.

References

[1] W. H. Kautz and R. C. Singleton, "Nonrandom binary superimposed codes," IEEE Trans. on Inf. Th., IT-10, No. 4, pp. 363-377, 1964.

[2] N. V. Semakov and V. A. Zinoviev, "Equal weight codes and tactical configurations," Prob. Pered. Info., Vol. 5, No. 3, pp. 28-36, 1969.

[3] T. Ericson, "Frequency hopping multiple access system and protocol sequences," DFVLR, IB 554-T-86/5, W. Germany, 1986.

[4] T. Ericson, "Bounds on the size of a code," Internal report, Linköping university, Sweden, 1986.

[5] F. J. MacWilliams and N. J. A. Sloane, The Theory of Error Correcting Codes , North Holland, 1977.

[6] A. G. Dyachkov and V. V. Rykov, "A survey of superimposed codes," Problems of Control and Information Theory, Vol. 12, No. 4, pp. 1-13, 1983.

[7] A. G. Dyachkov and V. V. Rykov, "Bounds on the length of disjunctive codes", Prob. Pered. Informatsii, Vol. 18, No. 3, pp. 166-171, English translation, 1981.

[8] G. D. Forney, Concatenated Codes, M. I. T. Press, Cambridge, MA., 1966.

[9] S. M. Johnson, "A new upper bound for error-correcting codes," IRE Trans. Inf. Th., Vol. IT-8, pp. 203-207, 1962.

[10] S. M. Johnson, "Upper bounds for consant weight error correcting codes," Discr. Math., Vol. 3, pp. 109-124, 1972.

[11] P. de Laval and S. Abdul-Jabbar, "Decoding of superimposed codes in multiaccess communication," Proc. of the Eurocon 88, Stockholm, June, 1988.