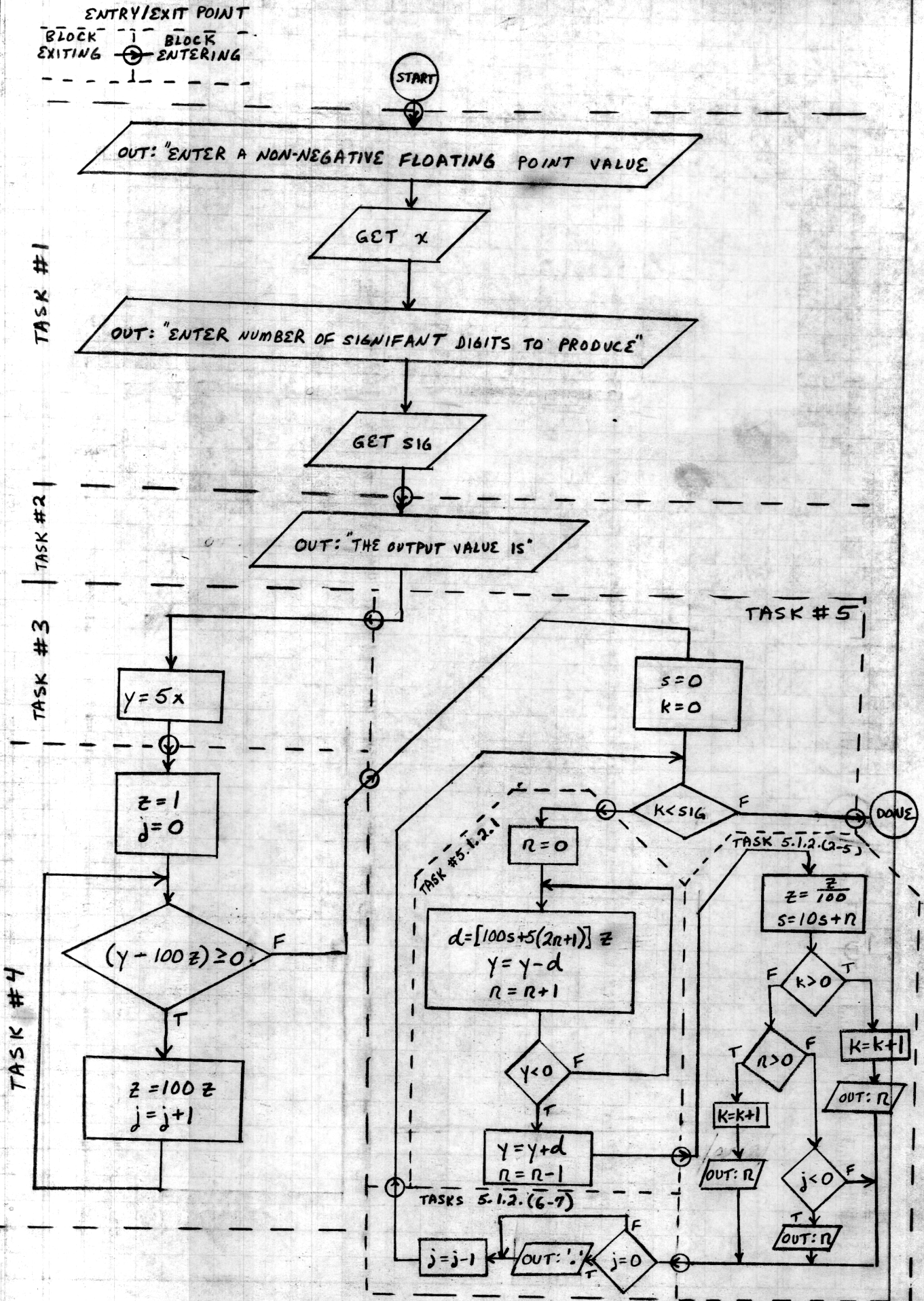




HAND EXAMPLES

$$x = 2$$

x	s16	y	z	j	s	k	n	d	
2	3	10	1	0					
				0	0				$(y - 100z) \geq 0?$ (F)
	5					0	5		$(k < s16)?$ (T)
						1			$(y < 0)?$ (F)
	-10					2	15		$(y < 0)?$ (T)
	5					1			
		0.01		1					$(k > 0)?$ (F) $(n > 0)?$ (T)
					1				$(j = 0)?$ (F)
									$(k < s16)?$ (T)
	3.95					0	1.05		
						1			$(y < 0)?$ (F)
	2.80					2	1.15		$(y < 0)?$ (F)
	1.55					3	1.25		" (F)
	0.20					4	1.35		" (F)
	-1.25					5	1.45		" (T)
	0.20					4			
		$\epsilon = 0.0001$		14					$(k > 0)?$ (T)
					2				$(j = 0)?$ (F)
									$(k < s16)?$ (F)
	0.0595					0	.1405		
	< 0					1	.1415		
						2			$(y < 0)?$ (T)
	0.0595	0.000001	141		1				$(k > 0)?$ (T)
									$(k < s16)?$ (F)
					-2	3			

OUT: '1'
OUT: '1'

OUT: '4'

OUT: '1'

OUTPUT: 1.41 (APPEARS TO BE $\sqrt{2}$)

INSTEAD OF BLINDLY WALKING THROUGH THE ALGORITHM STEP BY STEP FOR THE REMAINING VALUES, LET'S SEE IF WE CAN DESCRIBE THE ALGORITHM AT A LEVEL MORE APPROPRIATE FOR A HUMAN NOW THAT WE HAVE SEEN HOW IT WORKS ONCE.

OR, YOU CAN ALSO CRANK THROUGH BLINDLY.

HIGHER LEVEL DESCRIPTION

TASKS 1 THROUGH 3 - OBVIOUS

TASK 4: THIS TASK IS SEEING HOW MANY TIMES 100 WILL GO INTO y . THE FINAL VALUE OF z WILL BE THE LARGEST POWER OF 100 THAT IS STILL SMALLER THAN y . NOTICE THAT z & j ALWAYS CHANGE TOGETHER SO WE END UP WITH

$$z = 100^j$$

EXAMPLE: $y = 87654321$
 $j=3 \quad j=2 \quad j=1$

$$\therefore j=3; z=1,000,000$$

TASK 5: THE OUTER MOST LOOP JUST CYCLES UNTIL WE HAVE PRINTED THE REQUESTED NUMBER OF SIG FIGS. ON EACH PASS THERE ARE FOUR MAJOR SUBTASKS:

- (5A) TASK 5.1.2.1 DETERMINE NEXT DIGIT
- (5B) 5.1.2.(2-3) UPDATE z & s
- (5C) 5.1.2.(4-5) OUTPUT DIGIT (SUBJECT TO CONSTRAINTS)
- (5D) 5.1.2.(6-7) OUTPUT DECIMAL POINT (IF APPROPRIATE)

(5A) WE CAN SEE FROM THE HAND EXAMPLE THAT 'S' IS THE RESULT SO FAR AND THAT IT STARTS OUT AS ZERO.

WE THEN SHIFT THE RESULT SO FAR BY TWO PLACES AND PROGRESSIVELY ADD $(10R)+5$ AND SUBTRACT THE RESULT AS LONG AS y STAYS POSITIVE.

EXAMPLE: 87654321
 $n=0 \quad - 5 \quad (z=1,000,000)$
 $\quad \quad 82$
 $1 \quad - 15$
 $\quad \quad 67$
 $2 \quad - 25$
 $\quad \quad 42$
 $3 \quad - 35$
 $\quad \quad 7$
 $4 \quad - 45$
 $\quad \quad \leftarrow < 0$

SO THE FIRST DIGIT IS '4' AND THE NEW y IS 7,654,321

(5A) (CONT'D)

BEFORE WE COME BACK TO THIS TASK AGAIN, WE WILL DIVIDE Z BY 100 AND UPDATE OUR PARTIAL RESULT BY SHIFTING IT TO THE LEFT ONE PLACE AND TACKING ON THE DIGIT WE JUST IDENTIFIED. THESE ARE THE ONLY CHANGES TO THE VARIABLES THAT ARE USED IN THIS SUBTASK THAT WILL BE MADE PRIOR TO RE-ENTERING THIS SUBTASK

THE NEXT PASS THROUGH 5A WILL PRODUCE

$$\begin{array}{r}
 \begin{array}{r}
 r=0 \quad 7654321 \\
 \quad \quad 405 \\
 \hline
 \quad \quad 360 \\
 r=1 \quad 415 \\
 \hline
 \quad \quad <0
 \end{array}
 \quad
 \underbrace{z=10,000 \quad s=4}_{\text{THE RESULT OF TASK (5B)}}
 \end{array}$$

NEXT DIGIT IS '1', $y = 3604321$

NEXT PASS:

$$\begin{array}{r}
 3604321 \quad z=100 \quad s=41 \\
 - 4105 \\
 \hline
 31938 \\
 - 4115 \\
 \hline
 27823 \\
 - 4125 \\
 \hline
 23698 \\
 - 4135 \\
 \hline
 19563 \\
 - 4145 \\
 \hline
 15418 \\
 - 4155 \\
 \hline
 11263 \\
 - 4165 \\
 \hline
 7098 \\
 - 4175 \\
 \hline
 2923 \\
 - 4185 \\
 \hline
 <0
 \end{array}$$

'8' IS NEXT DIGIT.

TASK 5B: WE'VE ALREADY SEEN WHAT THIS TASK DOES.

TASK 5C: LOOK AT THE FOUR POSSIBLE PATHS THROUGH THIS TASK, WE SEE THAT THE ONLY TIME WE DO NOT INCREMENT K IS IF $(K=0)$ AND $(R=0)$. IN OTHER WORDS, IF WE HAVEN'T OUTPUT ANY SIGNIFICANT DIGITS AND THE PRESENT DIGIT IS ZERO (AND HENCE NOT SIGNIFICANT).

IF THE PRESENT DIGIT IS NOT SIGNIFICANT AND NO SIG FIGS HAVE YET BEEN PRINTED, WE WILL STILL PRINT A '0' IF $j < 0$. LOOKING BACK AT WHAT j IS, AND IF WE ARE RIGHT ABOUT THE FUNCTION BEING $\text{sqrt}()$, THE j IS INITIALLY SET TO THE NUMBER OF DIGITS TO THE LEFT OF THE DECIMAL POINT IN THE ANSWER. SO IF $j < 0$, WE ARE DEALING WITH A LEADING ZERO IN A NUMBER < 1 AND HENCE NEED TO PRINT IT.

TASK 5D: THIS TASK SIMPLY MONITORS ' j ' AND OUTPUTS A DECIMAL POINT AT THE PROPER TIME.

WITH THIS ANALYSIS IN MIND, WE CAN CARRY OUT THE ALGORITHM BY HAND MUCH MORE EFFICIENTLY.

$$X = 20,000$$

$$Y = 100,000$$

$$Z = 10000 \quad S = 0$$

$$\begin{array}{r} 5 \\ \hline 5 \\ \hline 105 \\ \hline 20 \end{array}$$

$$50000$$

$$S = 1 \quad Z = 100$$

} PRINT '1'

$$105$$

$$395$$

$$115$$

$$280$$

$$125$$

$$155$$

$$135$$

$$20$$

$$145$$

$$< 0$$

$$2000$$

$$S = 14 \quad Z = 1$$

} PRINT '4'

$$1405$$

$$595$$

$$1415$$

$$< 0$$

$$S = 141 \quad Z = 0$$

} PRINT '1'

$$x = 0.002$$

$$y = 0.010$$

$$s = 0 \quad z = 1$$

$$\begin{array}{r} \textcircled{0}5 \\ \hline < 0 \end{array}$$

> PRINT '0'

$$\begin{array}{r} 0.010 \\ \hline \end{array}$$

$$s = 0 \quad z = 0.01$$

$$\begin{array}{r} - \textcircled{0}5 \\ \hline \end{array}$$

$$s = 0 \quad z = 0.0001 \quad \text{PRINT '0'}$$

$$\begin{array}{r} < 0 \\ \hline \end{array}$$

$$0.01000$$

$$\begin{array}{r} 05 \\ \hline \end{array}$$

$$.0095$$

$$\begin{array}{r} 15 \\ \hline \end{array}$$

$$80$$

$$\begin{array}{r} 25 \\ \hline \end{array}$$

$$55$$

$$\begin{array}{r} 35 \\ \hline \end{array}$$

$$.0020$$

$$\begin{array}{r} .00\textcircled{4}5 \\ \hline \end{array}$$

$$s = 4 \quad z = 0.000001 \quad \text{PRINT '4'}$$

$$\begin{array}{r} < 0 \\ \hline \end{array}$$

$$.002000$$

$$\begin{array}{r} 405 \\ \hline \end{array}$$

$$1595$$

$$415$$

$$1180$$

$$425$$

$$755$$

$$435$$

$$320$$

$$445$$

$$\begin{array}{r} < 0 \\ \hline \end{array}$$

$$320\textcircled{00}$$

$$s = 44$$

PRINT '4'

$$\begin{array}{r} 4405 \\ \hline \end{array}$$

ADDING 2 ZEROS

$$27595$$

ALIGNS THE NUMBER TO

$$4415$$

MATCH THE SMALLER Z

$$23180$$

$$4425$$

$$18755$$

$$4435$$

$$14320$$

$$4445$$

$$9875$$

$$4455$$

$$5420$$

$$4465$$

$$955$$

$$\begin{array}{r} 44\textcircled{7}5 \\ \hline \end{array}$$

$$s = 447$$

PRINT '7'

$$< 0$$

RESULT IS 0.0447

FOR THE TWO ADDITIONAL VALUES, WILL USE $x = 100$ & $x = 90,000$
WITH THE EXPECTED RESULTS OF 10.0 AND 300

$$x = 100$$

$$y = 500 \quad s = 0 \quad z = 100 \quad j = 1$$

$$\begin{array}{r} 05 \\ \hline 0 \end{array}$$

$$0$$

$$\begin{array}{r} 15 \\ \hline <0 \end{array}$$

$$s = 1 \quad z = 1$$

$$j = 0$$

$$000$$

$$\begin{array}{r} -105 \\ \hline <0 \end{array}$$

$$s = 10 \quad z = 0.01$$

$$j = 1$$

$$<0$$

$$00000$$

$$\begin{array}{r} -105 \\ \hline <0 \end{array}$$

$$s = 100 \quad z = 0.0001$$

$$j = -1$$

RESULT IS 10.0

$$x = 90000$$

$$y = 450000 \quad s = 0 \quad z = 10000$$

$$\begin{array}{r} 5 \\ \hline 40 \end{array}$$

$$15$$

$$\begin{array}{r} 25 \\ \hline 25 \end{array}$$

$$0$$

$$0$$

$$\begin{array}{r} 35 \\ \hline <0 \end{array}$$

$$00000$$

$$s = 3 \quad z = 100$$

$$\begin{array}{r} 05 \\ \hline <0 \end{array}$$

$$s = 30 \quad z = 1$$

$$000$$

$$\begin{array}{r} 05 \\ \hline <0 \end{array}$$

$$<0$$

$$s = 300 \quad z = 0.01$$

RESULT IS 300