

NUMBER CONVERSIONSUNSIGNED INTEGERS1) 1,000,000 $\rightarrow$ hex

$$16 \overline{) 1,000,000}$$

$$62,500 \text{ R } 0$$

$$3906 \text{ R } 4$$

$$244 \text{ R } 2$$

$$15 \text{ R } 4$$

$$0 \text{ R } 15(F)$$

$$\underline{\underline{0xF4240}} \leftarrow \text{(#1)}$$

2) 0x8676 $\rightarrow$ dec

$$\begin{array}{r} 8 \\ \times 16 \\ \hline 128 \\ + 6 \\ \hline 134 \\ \times 16 \\ \hline 2144 \end{array} \quad \begin{array}{r} 2144 \\ + 7 \\ \hline 2151 \\ \times 16 \\ \hline 34,416 \\ + 6 \\ \hline 34,422 \end{array}$$

$$\underline{\underline{34,422}} \leftarrow \text{(#2)}$$

3) 1100 1011 0101₆ $\rightarrow$ dec
12 11 5 H

$$\begin{array}{r} 12 \\ \times 16 \\ \hline 192 \\ + 11 \\ \hline 203 \\ \times 16 \\ \hline 3248 \\ + 5 \\ \hline 3253 \end{array}$$

$$\underline{\underline{3253}} \leftarrow \text{(#3)}$$

#4) 47624₈ $\rightarrow$ dec

$$\begin{array}{r} 4 \\ \times 8 \\ \hline 32 \\ + 7 \\ \hline 39 \end{array} \quad \begin{array}{r} 39 \\ \times 8 \\ \hline 312 \\ + 6 \\ \hline 318 \end{array} \quad \begin{array}{r} 318 \\ \times 8 \\ \hline 2544 \\ + 2 \\ \hline 2546 \end{array} \quad \begin{array}{r} 2546 \\ \times 8 \\ \hline 20368 \\ + 4 \\ \hline 20372 \end{array}$$

$$\underline{\underline{20,372}} \leftarrow \text{(#4)}$$

NUMBER CONVERSIONS (CONT'D)UNSIGNED INTEGERS (CONT'D)

#5) 255 → bin

$$\begin{array}{r}
 16 \overline{) 255} \\
 15 \text{ r } 15 \text{ (F)} \\
 0 \text{ r } 15 \text{ (F)}
 \end{array}$$

0xFF ← (#5)

#6) 0x D7F5 → bin

(USING REF TABLE)

1101 0111 1111 0101 b ← (#6)

#7) 1101 0111 1111 0101 b → dec

13	→ 215	→ 3,455
<u>x16</u>	<u>x16</u>	<u>x16</u>
208	3,440	55,280
<u>+7</u>	<u>+15</u>	<u>+5</u>
215	3,455	55,285

55,285 ← (#7)#8) 625341₇ to base-5

TO BASE-10 USING REPEATED MULTIPLICATION : 107,535

TO BASE-5 51107535

$$\begin{array}{r}
 21507 \text{ R } 0 \\
 4301 \text{ R } 2 \\
 860 \text{ R } 1 \\
 172 \text{ R } 0 \\
 34 \text{ R } 2 \\
 6 \text{ R } 4 \\
 1 \text{ R } 1 \\
 0 \text{ R } 1
 \end{array}$$

1142 0120₅ ← (#8)

NUMBER CONVERSIONS (CONT'D)FIXED-POINT

#9) $\pi = 3.141592654$

base 10: 3.141593 ← (#9)

TO GET 6 radix places IN HEX

$$\pi = \frac{X}{16^6} \leftarrow \text{INTEGER.}$$

$$X = 16^6 \cdot \pi = 52707179_{10}$$

$$\begin{array}{r}
 16 \overline{) 52707179} \\
 \underline{3294198} \quad R \ 11 \ (B) \\
 205887 \quad R \ 6 \\
 \underline{12867} \quad R \ 15 \ (F) \\
 804 \quad R \ 3 \\
 \underline{50} \quad R \ 4 \\
 3 \quad R \ 2 \\
 \underline{0} \quad R \ 3
 \end{array}$$

base-16: 0x3.243F6B ←

TO GET BASE-2, TRANSLATE THE HEX

base-2: 11.001001b ←

#10) $e = 2.718281828$

base 10: 2.7183 ← (#10)

TO GET EQUIV. OF 4 DECIMAL PLACES, MULTIPLY AND DIVIDE
BY B^N SUCH THAT $10^4 < B^N$

$$\log_{10}(10^4) < \log_{10}(B^N)$$

$$4 \leq N \cdot \log_{10}(B)$$

$$N > \frac{4}{\log_{10}(B)}$$

FOR $B=16$, $N=4$

FOR $B=2$, $N=14$

TO GET 4 hex places: $X = 16^4 \cdot e = 178145_{10}$

$$\begin{array}{r}
 16 \overline{) 178145} \\
 \underline{11134} \quad R \ 1 \\
 695 \quad R \ 14 \ (E) \\
 \underline{43} \quad R \ 7 \\
 2 \quad R \ 11 \ (B) \\
 \underline{0} \quad R \ 2
 \end{array}$$

base-16: 0x2.B7E1 ←

base-2: 10.1011011111000b ←

NUMBER CONVERSIONS (CONT'D)

FLOATING-POINT VALUES (CONT'D)

$$\#11) 1 \text{ light-year} = 2.99 \times 10^8 \frac{\text{m}}{\text{s}} \cdot \frac{60 \text{s}}{1 \text{min}} \cdot \frac{60 \text{min}}{1 \text{hr}} \cdot \frac{24 \text{hr}}{1 \text{day}} \cdot \frac{365.25 \text{day}}{1 \text{yr}} \cdot 1 \text{yr}$$

$$1 \text{ ly} = 9.436 \times 10^{15} \text{ meters}$$

$$\text{base 10: } 9.44 \text{ e } 15 \leftarrow$$

(#11)

$$\text{hex } 16^N < 9.436 \times 10^{15} \leq 16^{N+1}$$

$$N+1 > \frac{\ln(9.436 \times 10^{15})}{\ln(16)} = 13.2 \quad (N=13)$$

$$\text{value} = M \cdot 16^{13}$$

$$M = \frac{\text{value}}{16^{13}} = 2.095$$

TO GET 0.01%, NEED

$$1 < (2.095)(0.01\%) \cdot 16^N$$

$$N > \frac{\ln\left(\frac{10000}{2.095}\right)}{\ln(16)} = 3.05 \quad (N=4)$$

$$\text{mantissa} \cdot 16^4 = 2.095 \times 16^4 = 137298$$

$$16 \overline{) 137298}$$

$$8581 \text{ r } 4$$

$$536 \text{ r } 5$$

$$33 \text{ r } 8$$

$$2 \text{ r } 1$$

$$0 \text{ r } 2$$

$$\text{base 16: } 0 \times 2.1854 \text{ e } D \leftarrow$$

$$\text{bin } 2.1854 \text{ e } D$$

$$10.0001 \ 1000 \ 0101 \ 0100 \text{ e } 1101$$

$$\underline{1.0000 \ 1100 \ 0010 \ 1010 \text{ e } 1110 \text{ b} \leftarrow}$$

NUMBER CONVERSIONS (CONT'D)FLOATING POINT VALUES (CONT'D)#12) CHARGE OF AN ELECTRON $|e^-| = -1.602 \times 10^{-19} \text{ C}$

$$0.1\% \rightarrow 1.602 \times 0.1\% = 0.0016$$

keep 3 decimal places

$$\text{base-10: } -1.602 \text{ e}^{-19} \leftarrow (\#12)$$

$$\text{hex: value} = \text{mantissa} \times 16^N = 1.602 \text{ e}^{-19} \text{ (Abs value)}$$

$$\text{mantissa} = \frac{\text{value}}{16^N}$$

$$1.0 \leq \text{mantissa} < 16$$

$$1.0 \leq \frac{\text{value}}{16^N} < 16$$

$$16^N \leq \text{value}$$

$$N \ln(16) \leq \ln(\text{value})$$

$$N \leq \frac{\ln(\text{value})}{\ln(16)} = -15.6 \Rightarrow N = -16$$

$$\text{mantissa} = \frac{\text{value}}{16^{-16}} = \frac{1.602 \times 10^{-19}}{16^{-16}} = 2.9552 \text{ (0.1\% = 0.003)}$$

$$\frac{\text{mantissa} \cdot 16^3}{16^3} = \frac{12104}{16^3}$$

$$16 \overline{) 12104}$$

$$\left. \begin{array}{r} 756 \text{ r } 8 \\ 47 \text{ r } 4 \\ 2 \text{ r } F \\ 0 \text{ r } 2 \end{array} \right\} \frac{0x2F48}{16^3} = 0x2.F48$$

$$\text{base-16: } -0x2.F48 \text{ e}^{-10} \leftarrow$$

$$\text{bin: } -0x2.F48 \text{ e}^{-10}$$

$$-10.111101001000 \text{ e}^{-10000} \text{ b}$$

$$\text{base-2: } -1.011101001 \text{ b} \leftarrow$$

ARITHMETIC IN OTHER NUMBER BASESHEXADECIMAL ARITHMETIC TABLES

#13 PREPARE HEX ADDITION & MULTIPLICATION TABLES

HEXADECIMAL ADDITION TABLE

+	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
1	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F	10
2	2	3	4	5	6	7	8	9	A	B	C	D	E	F	10	11
3	3	4	5	6	7	8	9	A	B	C	D	E	F	10	11	12
4	4	5	6	7	8	9	A	B	C	D	E	F	10	11	12	13
5	5	6	7	8	9	A	B	C	D	E	F	10	11	12	13	14
6	6	7	8	9	A	B	C	D	E	F	10	11	12	13	14	15
7	7	8	9	A	B	C	D	E	F	10	11	12	13	14	15	16
8	8	9	A	B	C	D	E	F	10	11	12	13	14	15	16	17
9	9	A	B	C	D	E	F	10	11	12	13	14	15	16	17	18
A	A	B	C	D	E	F	10	11	12	13	14	15	16	17	18	19
B	B	C	D	E	F	10	11	12	13	14	15	16	17	18	19	1A
C	C	D	E	F	10	11	12	13	14	15	16	17	18	19	1A	1B
D	D	E	F	10	11	12	13	14	15	16	17	18	19	1A	1B	1C
E	E	F	10	11	12	13	14	15	16	17	18	19	1A	1B	1C	1D
F	F	10	11	12	13	14	15	16	17	18	19	1A	1B	1C	1D	1E

HEXADECIMAL MULTIPLICATION TABLE

X	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
2	0	2	4	6	8	A	C	E	10	12	14	16	18	1A	1C	1E
3	0	3	6	9	C	F	12	15	18	1B	1E	21	24	27	2A	2D
4	0	4	8	C	10	14	18	1C	20	24	28	2C	30	34	38	3C
5	0	5	A	F	14	19	1E	23	28	2D	32	37	3C	41	46	4B
6	0	6	C	12	18	1E	24	2A	30	36	3C	42	48	4E	54	5A
7	0	7	E	15	1C	23	2A	31	38	3F	46	4D	54	5B	62	69
8	0	8	10	18	20	28	30	38	40	48	50	58	60	68	70	78
9	0	9	12	1B	24	2D	36	3F	48	51	5A	63	6C	75	7E	87
A	0	A	14	1E	28	32	3C	46	50	5A	64	6E	78	82	8C	96
B	0	B	16	21	2C	37	42	4D	58	63	6E	79	84	8F	9A	A5
C	0	C	18	24	30	3C	48	54	60	6C	78	84	90	9C	A8	B4
D	0	D	1A	27	34	41	4E	5B	68	75	82	8F	9C	A9	B6	C3
E	0	E	1C	2A	38	46	54	62	70	7E	8C	9A	A8	B6	C4	D2
F	0	F	1E	2D	3C	4B	5A	69	78	87	96	A5	B4	C3	D2	E1

HEXADECIMAL ARITHMETIC

#14) CB
+ 1D

E8

0xE8 ← (#14)

#15) $\begin{array}{r} CB \\ -1D \\ \hline AE \end{array}$

$O \times AE$ ← (#15)

#16)

$$\begin{array}{r} CB \\ \times 1D \\ \hline A4F \\ CB \\ \hline 16FF \end{array}$$

0x16FF ← (#16)

#17) $1D \overline{) CB}$ $\begin{array}{r} 7r0 \\ CB \end{array}$ $\begin{array}{r} 1D \\ \times 6 \\ \hline AE \end{array}$ $\begin{array}{r} 1D \\ \times 7 \\ \hline CB \end{array}$

$0x7r0 \leftarrow$ (#17)
 $0x7.00 \leftarrow$

#18) 'FACE
+ BEAD
18978

0x1B97B ← (#18)

#19)
$$\begin{array}{r} \text{FACE} \\ - \text{BEAD} \\ \hline 3C21 \end{array}$$

0x3C21 ← (#19)

#20) FACE

$$\begin{array}{r} \times B E A D \\ \hline \end{array}$$

$$\begin{array}{r} \begin{array}{ccccccc} & & & & & & C \\ & & & & & & +C \\ & & & & & 18 \\ & & & & & +4 \\ & & & & & \hline & & & & & 1C \\ & & & & & +A \\ & & & & & \hline & & & & & 26 \end{array} \\ \begin{array}{ccccccc} & & & & & & 16 \\ & & & & & & 18 \\ & & & & & 2E \\ & & & & & \hline & & & & & 16 \\ & & & & & 18 \\ & & & & & +4 \\ & & & & & \hline & & & & & 1C \\ & & & & & +A \\ & & & & & \hline & & & & & 26 \end{array} \end{array}$$

0x BACE6136 ← (#20)

#21) BEAD

FACE	1.50	BEAD	BEAD
BEAD	$\times 4$		$\times 5$
	2.00	2 FAC 4	73.71
3C210			
38971			
<u>389F00</u>			

$0x1r0x3C21 \leftarrow (\#21)$
 $0x1.51 \leftarrow$

COMPUTER BASED REPRESENTATIONSNIBBLES AND BITS

#22 FOR N-bits IN 2's complement

 $N=4$

$$\text{MAX } 0111 = 1000 - 1 = 2^3 - 1 \quad \text{GENERAL } 2^{(N-1)} - 1$$

$$\text{MIN } 1000 = 1001 - 1 = -(2^3 - 1) \quad \text{GENERAL } -2^{(N-1)}$$

SIZE	BITS	MIN	MAX
byte	8	$-2^7 = -128$	$+127$
word	16	$-2^{15} = -32,768$	$+32,767$
dword	32	$-2^{31} = -2,147,483,648$	$+2,147,483,647$

} (#22)

#23 $1610,000 = 0x2710$ $161625 r0$ $16139 r1$ $12 r7$ $0 r2$ OFFSET $\text{NUM} + 0x8000$

SIGNED 0010|0111|0001|0000 0x2710

OFFSET 1010|0111|0001|0000 0xA710

2's comp 0010|0111|0001|0000 0x2710

} (#23)

#24 OFFSET $-10,000$

1000 0000 0000 0000

-0010 0111 0001 0000

0101 1000 1111 0000

 $\sim 0x2710 \Rightarrow 0xD8EF$ $+1$

0xD8F0

SIGNED 1010|0111|0001|0000 0xA710

OFFSET 0101|1000|1111|0000 0x58F0

2's comp 1101|1000|1111|0000 0xD8F0

} (#24)

#25 $+1$

SIGNED 0000|0000|0000|0001 0x0001

OFFSET 1000|0000|0000|0001 0x8001

2's comp 0000|0000|0000|0001 0x0001

} (#25)

#26 -1

SIGNED 1000|0000|0000|0001 0x8001

OFFSET 0111|1111|1111|1111 0x7FFF

2's comp 1111|1111|1111|1111 0xFFFF

} (#26)

#27 $-32,767$ 16132767 $2047 r15$ $127 r15$ $7 r15$ $0 r7$

SIGNED 1111|1111|1111|1111 0xFFFF

OFFSET 0000|0000|0000|0001 0x0001

2's comp 1000|0000|0000|0001 0x8001

} (#27)

NOTICE THE PATTERN - OFFSET BINARY IS 2's COMP W/ SIGN BIT FLIPPED
(WHEN 1000... IS "0" IN OFFSET BINARY)

COMPUTER BASED REPRESENTATIONS (CONT'D)SIGNED INTEGERS (CONT'D)

#28) 0xFFFF (NEGATIVE)

-1 ← (#28)

$$\begin{array}{r} \sim(\text{FFFF}) = 0x0000 \\ +1 \quad \quad +1 \\ \hline 0x0001 \Rightarrow 1 \end{array}$$

#29) 0x8000 (NEGATIVE)

-32,768 ← (#29)

$$\begin{array}{r} \sim(0x8000) = 0x7FFF \\ +1 \quad \quad +1 \\ \hline 0x8000 \Rightarrow 2^{15} = 32,768 \end{array}$$

NOTE: THIS IS THE "TROUBLESOME" VALUE

#30) 0x7FFF (POSITIVE)

32,767 ← (#30)

$$\begin{array}{r} = 0x8000 = 2^{15} \\ -1 \quad -1 \\ \hline 32,767 \end{array}$$

#31) 0x0000

0 ← (#31)

#32) 0xBEAD (NEGATIVE)

-16723 ← (#32)

$$\begin{array}{r} \sim(0xBEAD) = 0x4152 \\ +1 \quad \quad +1 \\ \hline 0x4153 \Rightarrow 16723 \\ \text{(by repeated multiplication)} \end{array}$$

#33) 0xFACE (NEGATIVE)

-1330 ← (#33)

$$\begin{array}{r} \sim(0xFACE) = 0x0531 \\ +1 \quad \quad +1 \\ \hline 0x0532 \Rightarrow 1330 \end{array}$$

#34) 0x6666 (POSITIVE)

26,214 ← (#34)REPEATED MULTIPLICATION $\Rightarrow 26,214$

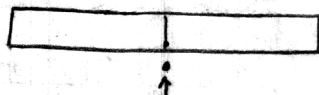
#35) 0x5000 (POSITIVE)

20,480 ← (#35)

$$5 \times 16^3 = 20,480$$

COMPUTER BASED REPRESENTATIONS (CONT'D)FIXED POINT REPRESENTATION

#36



$$\text{LARGEST POSITIVE VALUE} = 0x7F.FF = 127 + \frac{255}{256} = 127.996 \leftarrow (\#36)$$

$$\#37 \text{ SMALLEST POSITIVE VALUE} = 0x00.01 = 0 + \frac{1}{256} = 0.0039 \leftarrow (\#37)$$

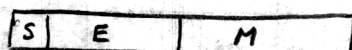
#38

1111 1111.1111 1111 (NEGATIVE)
 0000 0000.0000 0000 (INVERT)
 0000 0000.0000 0001 (+1)
 0x00.01 (ABS. VALUE)

$$= -0.0039 \leftarrow (\#38)$$

FLOATING POINT REPRESENTATION

IEEE SINGLE PRECISION: 32 bits w/ 8 bit exponent



$$\text{value} = (-1)^S \cdot m \cdot 2^E$$

$$\text{for } E > 0 : e = E - 127$$

$$m = 1 + \frac{M}{2^{23}}$$

$$E = 0 : e = -126$$

$$m = \frac{M}{2^{23}}$$

$$\#39 \text{ SMALLEST value : } S=0, E=0, M=1$$

$$\text{value} = \frac{1}{2^{23}} \cdot 2^{-126} = 2^{-149} = 1.401 \times 10^{-45} \leftarrow (\#39)$$

$$\#40 \text{ largest value : } S=0, E=0xFE, M=\text{all 1's}$$

$$\text{value} \approx 2 \times 2^{127} = 2^{128} = 3.402 \times 10^{38} \leftarrow (\#40)$$

$$\#41 \text{ smallest } x \text{ for } 1+x : S=0, E=0x7F, M=0$$

$$1+x : S=0, E=0x7F, M=1$$

$$x = \frac{1}{2^{23}} = 1.2 \times 10^{-7} \leftarrow (\#41)$$

$$\#42 \text{ static range } \frac{\text{largest}}{\text{smallest}} = \frac{3.402 \times 10^{38}}{1.401 \times 10^{-45}} = 2.4 \times 10^{83} \leftarrow (\#42)$$

$$\#43 \text{ dynamic range } \frac{1}{x} = \frac{1}{1.2 \times 10^{-7}} = 8.3 \times 10^6 \leftarrow (\#43)$$

COMPUTER-BASED REPRESENTATION (CONT'D)FLOATING POINT REPRESENTATION (CONT'D)

#44) sig figs $\log_{10}(\text{dynamic range}) = \underline{6.9}$ (#44)

#45) largest contiguous R: $2^{24} = \underline{16,777,216}$ (#45)

(USING REASONING FOR DOUBLE PRECISION FROM MODULE)

#46) SPEED OF LIGHT, $c = 299,792,458 \text{ m/s}$ (EXACT IN VACUUM)

$$C = \left(1 + \frac{M}{2^{23}}\right) 2^e \leftarrow \text{exponent, not } e = 2.718...$$

$$2^e \leq C < 2^{(e+1)}$$

$$e \ln(2) = \ln(C)$$

$$e = \frac{\ln(C)}{\ln(2)} = 28.6 \quad \underline{e=28} = E-127 \therefore E=155 \quad (0x9B)$$

$$M = 2^{23} \left(\frac{C}{2^e} - 1\right) = 2^{23} (0.1168139) = 979.906$$

to hex: 16|979906

$$61244 \text{ r } 2$$

$$3827 \text{ r } 12 (C)$$

$$239 \text{ r } 3$$

$$14 \text{ r } 15 (F)$$

$$0 \text{ r } 14 (E)$$

$$\begin{array}{c} 000 \ 1110 \ 1111 \ 0011 \ 1100 \ 0010 \\ \hline 20 \\ \hline 23 \end{array}$$

$$\begin{array}{c} S \quad E \quad M \\ 0 \ 1001 \ 1011 \ 000 \ 1110 \ 1111 \ 0011 \ 1100 \ 0010 \\ \hline 0100 \ 1101 \ 1000 \ 1110 \ 1111 \ 0011 \ 1100 \ 0010 \\ 4 \ D \ 8 \ E \ F \ 3 \ C \ 2 \end{array}$$

0x4D8EF3C2 (#46)

COMPUTER BASED REPRESENTATIONS (CONT'D)CHARACTERS AND STRINGS

#47	0xC000	'E'	0x45
	C001	'C'	43
	C002	'E'	45
	C003	'.'	2D
	C004	'I'	31
	C005	'O'	30
	C006	'2'	32
	C007	'I'	31
	C008	':'	3A
	C009	SP	20
	C00A	'H'	48
	C00B	'o'	6F
	C00C	'm'	6D
	C00D	'e'	65
	C00E	'w'	77
	C00F	'o'	6F
	C010	'r'	72
	C011	'k'	6B
	C012	SP	20
	C013	'#'	23
	C014	'1'	31
	C015	NUL	00

← (#47)

BYTE ORDERING AND CROSS REPRESENTATION

#48	0xDFF0	'H'	48
	DFF1	'e'	66
	DFF2	'I'	6C
	DFF3	'I'	6C
	DFF4	'O'	6F
	DFF5	NUL	00
	DFF6	(-10,000)	FO
	DFF7	(PROB #24)	D8
	DFF8	(IT)	DA
	DFF9	(FROM)	0F
	DFFA	(MODULE)	49
	DFFB		40
	DFFC	4000000000	00
	DFFD	250,000,000 r0	28
	DFFE	15,625,000 r0	6B
	DFFF	976562 r8	EE
		61,035 r2	
		3814 rB	
		238 r6	
		14 rE	
		0 rE	

← (#48)

#49) 0xDFFC

EE6B2800

11011100 110182800

- DC 682800

220 0x6B2800

-127 1+ 2²³

93

-1.837158 × 2⁹³-1.819433 × 10²⁸

← (#49)

#50) 0xDFFD C6C

0x48666C6C

1,214,671,980 ← (#50)