Sec: 1 2 (circle)	ECE-1021 Fall 2004 Exam #2	02 November 2004
NAME:		Score:/100
Unless specified other	wise, assume that:	
<ul><li>The variables u,</li><li>Any reference to</li></ul>	j, k, l, m, and n are declared as int v, w, x, y, and z are declared as doubles. a "floating point" value is to be taken as a type an "integer" value is to be taken as a type	a type double.
IEEE-574 Single-precis IEEE-574 Double-preci	ion floating point standard: 32 bits total in sion floating point standard: 64 bits total i	cluding an 8-bit exponent. ncluding an 11-bit exponen
Multiple Choice (2 poin	ats each) - choose the best answer from an	nong those offered.
1) How many bits are	used by the standard ASCII code?	
a) 4. 7. c) 8. d) 16.		
2) All ASCII character	s fall into exactly one of which of the follo	owing pairs of groups?
<ul><li>Printing and Cont</li><li>Upper case and I</li><li>Alphanumeric and</li><li>Control and Gra</li></ul>	Lower case. nd Punctuation.	
3) What is the maximu	m number of distinct values that can be re	presented with 32 bits?
4,294,967,296 b) 2,147,483,648 c) 65,536 d) 32,768		
4) What must be true of	f a switch() statement?	
그는 그는 그를 가장 하는 사람들이 되는 것이 되는 것이 되는 것이 되었다. 이렇게 하면 하는데 하는데 하는데 되었다.	expression must return an integer result.  s required and must appear last in the case	list.

c) The only way to exit a switch() structure is through the use of a break statment.

d) Multiple values can be listed on a case line – such as case 1,2,3:

a) A logical operator will always return a value of 0 or 1.

Negative values cannot be interpreted as logical values.

b) A value is a logical FALSE if it is equal to 0.

d) A value is a logical TRUE if it is equal to -1.

5) Which of the following statements is false?

- 6) Which of the following is not one of the allowed ways to represent negative integers under the C Language Standard?
  - Offset binary.
  - b) Signed binary.
  - c) One's complement.
  - d) Two's complement
- 7) A four-byte integer is stored in addresses 0x4BFC through 0x4BFF with the most significant byte at location 0x4BFC. What is true about this representation?
  - a) It is stored at 0x4BFF in Big Endian format.
  - b) It is stored at 0x4BFF in Little Endian format.
  - It is stored at 0x4BFC in Big Endian format.
  - d) It is stored at 0x4BFC in Little Endian format.
- 8) In the base-5 number 323.041, what is the weighting of the digit '4'?
  - 0.04

$$\frac{1}{5^2} = \frac{1}{25} = 0.04$$

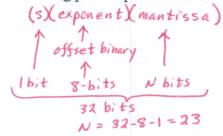
- b) 1/20
- c) 5e-1
- d) 5e-3
- 9) If it takes N digits to represent the integer value V in a positional number system using base B, which of the following relations will be true? 2 DIGITS IN BASE 10
  - a)  $V = B^N$

b)  $(B^N < V)$  AND  $(V \le B^(N+1))$ 

- d)  $V^N = B$
- 10) Which of the following is not a requirement of the case labels in a switch() construct?
  - a) They must unique.
  - They must evaluate to valid ASCII codes.
  - c) They must be integers expressions.
  - d) They must be constant expressions.

Questions 11-15 refer to an IEEE-754 Single Precision floating point representation.

- 11) What representation is used for the exponent?
  - a) signed binary.
  - offset binary.
  - c) one's complement.
  - d) two's complement.



- 12) How many bits of the mantissa are stored?
  - a) 16.
  - **b)** 23.
  - c) 24.
  - d) 33.
- 13) In what order are the components that make up the value stored?
  - (sign)(exponent)(mantissa)
  - b) (sign)(mantissa)(exponent)
  - c) (exponent)(sign)(mantissa)
  - d) (mantissa)(exponent)(sign)
- 14) If the representation for positive infinity were stored in Little Endian format, what would be stored in the four bytes, beginning with the base address?
  - **(8)** [00][00][80][7F]
  - b) [00][00][00][FF]
  - c) [7F][80][00][00]
  - d) [FF][FF][FF]

LITTLE ENDIAN : 00 00 90 90 00 00 80 7F

- 15) A floating point value is represented by the pattern 0xBEC00000. What is the value in decimal?
  - a)  $-0.75 \times 2^{-16}$ .
  - **⑥** −0.375.
  - c) +0.375.
  - d)  $+0.75 \times 2^{16}$ .

B E CO OO OO

Negative exp = 70 mantissa = 
$$1.1 \leftarrow 5ixll$$
 point binary

base ten

125

actual exponent

125-127 = -2

 $V = -(11 \times 2)(2^{2})$ 
 $-3 \times 2^{3} = -\frac{3}{8} = -.375$ 

- 16) Repeated division by the number base is primarily useful for what purpose?
  - Onverting a number from base-10 to another base.
  - b) Converting a number between two non-decimal number bases.
  - c) Converting a number from another base to base-10.
  - d) Determining how many decimal digits a number has.
- 17) Repeated multiplication by the number base is primarily useful for what purpose?
  - a) Converting a number from base-10 to another base.
  - b) Converting a number between two non-decimal number bases.
  - Converting a number from another base to base-10.
  - d) Determining how many decimal digits a number has.
- 18) How is zero represented in an IEEE-754 standard floating point representation?
  - As a pattern of all zeros.
  - b) Zero cannot be exactly represented because of the implied leading one in the mantissa.
  - c) As a pattern with a one as the second bit from the left.
  - d) As a pattern of all zeros except for a leading one.
- 19) In an IEEE-754 representation, why does an exponent consisting of all zeros and an exponent consisting of all zeros except for a trailing one represent the same exponent value?
  - To avoid a gap as the representation moves from a normalized to a non-normalized interpretation.
  - b) To increase the static range of the representation.
  - c) So that zero may be exactly represented.
  - d) It doesn't.
- 20) Given a bit pattern representing a floating point value in the standard IEEE format, how can the negative of that value be represented?
  - a) By taking the two's complement of the mantissa.
  - b) By inverting the sign bit and all bits of the mantissa.
  - By treating the entire pattern as though it were an integer and taking the two's complement of it.
  - By inverting the sign bit.

Short Answer (2 pts each)

Questions 16-20 refer to the speed of light in a vacuum which is exactly c = 299,792,458 m/s.

21) How many digits are required to express this in base 7?

$$7 \le c \le 7$$

(NE)  $f_{n}(7) \le |n(c)|$ 

(NE)  $\frac{|n(c)|}{|n(7)|} = 10.03 + (= 11.03)$ 
 $A = 11$ 



22) What is the smallest base that could express this value in six digits or less?

$$\beta^{N-1} \leq C < \beta^{N} \quad N = 6$$

$$C < \beta^{6} \quad |n(\beta)| > \frac{|n(c)|}{6}$$

$$B > C < \frac{|n(c)|}{6} = 25.86$$

$$B = 26$$

23) How many bits are required to express this value?



24) How many digits would the Babylonians have used in their base-60 numbering system?

$$\frac{1n(c)}{In(60)} < N => N > 4.76$$

$$N = 5$$



25) How many hexadecimal digits are required to represent c<sup>2</sup> (the square of the speed of light)?

$$c^{2} < 16^{N}$$

$$2 \ln(c) < N \ln(16)$$

$$N > 2 \cdot \frac{\ln(c)}{\ln(16)} = 14.07$$

$$N = 15$$



Longer Answer (10 pts each)

26) Fill in the following table with the decimal value represented by each bit patterns using the representation at the top of the column.

PATTERN	Pure Binary	Signed Binary	Offset Binary	One's Comp	Two's Compl		
000	0	0	-4	0	0		
001	1	1	-3	1	1		
010	2	2	-2	2	2		
011	3		-1	3	3		
100	4	-0	0	-3	-4		
101	5	-1	1	-2	-3		
110	6	-2	2	-1	-2		
111	7	-3	3	-0	-1		

27) Performing all computations directly in hexadecimal, write the result of 0xFADE divided by 0x0BAD in quotient-remainder form (i.e., the quotient is an integer).

28) (10 pts) A particular number is represented using two bytes in signed binary, offset binary, and two's compliment format. In no particular order, the three representations yield values of 0xA710, 0x83F0, and 0x58F0. What is the original base-10 number?

		EN BIT		
value	SIGNED	OFFSET	TWO'S	
>0	0	-1	Ó	
<0		0	1	
A710 - 83FØ - 58FØ-	→ SIGN B → SIGN BI	T=1  $ T=1 $ $ T=0 $	NEGATIU	FFSET BINARY

OFFSET BINARY;

KALLE = REPRESENTATION - OFFSET

OFFSET = 8000

-VALUE = OFFSET - REPRESENTATION

VALUE = ( 0x 2710

```
×16

32

+7

39

×16

234

39

624

+1

625

×16

3750

625

10000
```

29) (10 pts) Write a complete C function called log\_b() that takes two floating point arguments. The function returns the logarithm of the first number using the second number as the base. The base is assumed to be strictly positive.



$$|n(x) = \gamma(|n(b))$$

$$\gamma = \frac{\ln(x)}{\ln(b)}$$

#include <math.h> /\* log() \*/

double log\_b (double x, double b)

{
return log(x)/log(b);

36:08

30) (10 pts) Without using the pow() function, write a complete C function called YrootofX() that takes two floating point arguments. The function returns the yth root of x where x is the first argument (assumed to be strictly positive) and y is the second argument (assumed to be non-zero).

Example: z = YrootofX(64, 3); /\* Should store the value 4 in the variable z \*/

$$y^{2} = 64$$

$$y^{2} = 7$$

$$y = 7$$

$$|n(y) = |n(x)|$$

$$|n(y) = \frac{|n(x)|}{root}$$

$$y = exp(\frac{|n(x)|}{root})$$

#include < math. h > /\* log(), exp() \*/

double Yrootofx (double x, double root)

{

return (exp (log(x)/root));

# 31) EXTRA CREDIT #1: (10 pts)

A sign board has six tri-color lights. Each light can display one of three colors – red, green, or blue. The value of each color is as follows: red = 0; green = 1; blue = 2.

a) How many different values can be represented by this sign board? 729

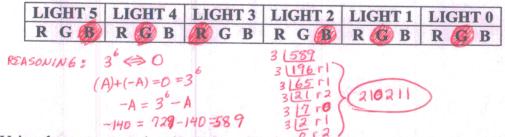
b) If the lights represent an unsigned integer according to a conventional positional numbering system, what decimal value is represented by the following configuration of lights?

LIGHT 5 BLUE	LIGHT 4 GREEN	LIGHT 3 RED	LIGHT 2 BLUE	LIGHT 1 RED	LIGHT 0 BLUE
2 2×3=6-	1 → 5+6	O 12+€ 1	2 →+63	O +195 F	2
	7 ×3 21	20 x3 63	65 x3 195	195 ×3 585	587

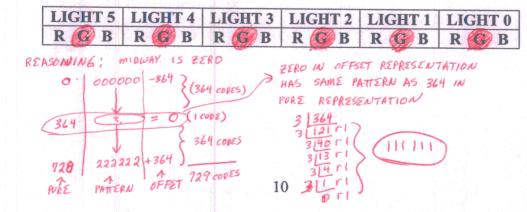
c) Circle the color of each light if the decimal value of the number being displayed 140.

LIGHT 5	LIGHT 4		LIGHT 3		LIGHT 2		LIGHT 1		LIGHT 0			
R) G B	R	G B	R	G B	R	G	B	R	G	В	R	GB
3 [ 140												
3146 12	7	12012										
315 10	2	ia - i -										
211 12												

d) Using the same reasoning that led to 2's compliment for representing signed values in binary, what color would each light be if -140 is displayed using 3's compliment?



e) Using the same reasoning that led to offset binary for representing signed values in binary, what color would each light be for the value zero?



## EXTRA CREDIT #2 (10 pts)

For large values of n, the factorial of n can be closely approximated by the following equation:

$$n! = \sqrt{2\pi n} \left(\frac{n}{e}\right)^n$$

For instance, for n = 10 the above equation, known as the Stirling Formula, yields a value that is within 1% of the correct answer.

(5 pts) Write a complete C function called fact\_digits() that takes a double n as the argument and returns a floating point (double) value that is the number of digits in n!. Your function needs to be able to accept values of n for which n! would be far too large to accommodate even in a variable of long double. For instance, the user might wish to use your function to discover what the first value of n! is that has more than one trillion (10<sup>12</sup>) digits. A long double, on most compilers, can't even represent a number with 15,000 digits.

Hint: How many digits are in 12345? What is the base ten log of 12345?  $log_{10}(12345) = 4.09149$  digits =  $sloor(log_{10}(12345) + 1)$  (5 pts) Within 1%, what is the first value of n for which n! has more than a thousand digits?

Hint: You can do this reasonably quickly on your calculator - but be sure to be finished with the rest of the exam first. Show your work for full credit.

How many digitis in y? digits = floor (logio (y)+1) logio(V!) =0.5.logio (fam) + n logio (n/e) #include <math.h ) /\* log10(), fdoor() \*/ #define PI (3.1415926) double fact\_digits (double n) double digits; digits = R\* log 10 (R/ exp(1)); digits += 0.5 \* log 10 (2.0\*PI\*n); return floor (digits+1); FOR 450 0.5 logio (2mr) = 1,7218 1000.23 21 DIGITS SHORT 450 -> 1001 digits 11 449 -> 998 digits 1:06:21