Sec: 1 2 (circle) ECE-1021 Fall 2004 Exam #2 02 November 2004

NAME:____________________________ Score:______/100

Unless specified otherwise, assume that:

- The variables i, j, k, l, m, and n are declared as int
- The variables u, v, w, x, y, and z are declared as doubles.
- Any reference to a "floating point" value is to be taken as a type double.
- Any reference to an "integer" value is to be taken as a type int.

IEEE-574 Single-precision floating point standard: 32 bits total including an 8-bit exponent.
IEEE-574 Double-precision floating point standard: 64 bits total including an 11-bit exponent.

Multiple Choice (2 points each) – choose the best answer from among those offered.

1) How many bits are used by the standard ASCII code?

   a) 4.
   b) 7.
   c) 8.
   d) 16.

2) All ASCII characters fall into exactly one of which of the following pairs of groups?

   a) Printing and Control.
   b) Upper case and Lower case.
   c) Alphanumeric and Punctuation.
   d) Control and Graphical.

3) What is the maximum number of distinct values that can be represented with 32 bits?

   a) 4,294,967,296
   b) 2,147,483,648
   c) 65,536
   d) 32,768

4) What must be true of a *switch()* statement?

   a) The controlling expression must return an integer result.
   b) A default *case* is required and must appear last in the *case* list.
   c) The only way to exit a *switch()* structure is through the use of a break statment.
   d) Multiple values can be listed on a case line – such as *case 1,2,3*:

5) Which of the following statements is false?

   a) A logical operator will always return a value of 0 or 1.
   b) A value is a logical FALSE if it is equal to 0.
   c) Negative values cannot be interpreted as logical values.
   d) A value is a logical TRUE if it is equal to –1.

6) Which of the following is not one of the allowed ways to represent negative integers under the C Language Standard?

   a) Offset binary.
   b) Signed binary.
   c) One's complement.
   d) Two's complement

7) A four-byte integer is stored in addresses 0x4BFC through 0x4BFF with the most significant byte at location 0x4BFC. What is true about this representation?

   a) It is stored at 0x4BFF in Big Endian format.
   b) It is stored at 0x4BFF in Little Endian format.
   c) It is stored at 0x4BFC in Big Endian format.
   d) It is stored at 0x4BFC in Little Endian format.

8) In the base-5 number 323.041, what is the weighting of the digit '4'?

   a) 0.04
   b) 1/20
   c) 5e-1
   d) 5e-3

$\frac{1}{5^2} = \frac{1}{25} = 0.04$

9) If it takes N digits to represent the integer value V in a positional number system using base B, which of the following relations will be true?

   a) V = B^N
   b) (B^N < V) AND (V <= B^(N+1))
   c) (B^(N-1) <= V) AND (V < B^N)
   d) V^N = B

3 DIGITS IN BASE 10

$100 \leq abc \leq 999 < 1000$

$10^2 \leq abc < 10^3$

$B^{N-1} \leq V < B^N$

10) Which of the following is not a requirement of the *case* labels in a *switch()* construct?

   a) They must unique.
   b) They must evaluate to valid ASCII codes.
   c) They must be integers expressions.
   d) They must be constant expressions.

Questions 11-15 refer to an IEEE-754 Single Precision floating point representation.

11) What representation is used for the exponent?

a) signed binary.
b) offset binary.
c) one's complement.
d) two's complement.

(s)(exponent)(mantissa)
offset binary
1 bit 8-bits N bits
$\underbrace{\text{1 bit} \quad \text{8-bits} \quad \text{N bits}}_{\text{32 bits}}$
$N = 32-8-1=23$

12) How many bits of the mantissa are stored?

a) 16.
b) 23.
c) 24.
d) 33.

13) In what order are the components that make up the value stored?

a) (sign)(exponent)(mantissa)
b) (sign)(mantissa)(exponent)
c) (exponent)(sign)(mantissa)
d) (mantissa)(exponent)(sign)

14) If the representation for positive infinity were stored in Little Endian format, what would be stored in the four bytes, beginning with the base address?

a) [00][00][80][7F]
b) [00][00][00][FF]
c) [7F][80][00][00]
d) [FF][FF][FF][FF]

(s)(exponent)(mantissa)
(0)(1111 1111)(000,.....00)
7F | 80 | 00 | 00

LITTLE ENDIAN: ~~00 00 80 7F~~ 00 00 80 7F

15) A floating point value is represented by the pattern 0xBEC00000. What is the value in decimal?

a) -0.75×2^{-16}.
b) -0.375.
c) $+0.375$.
d) $+0.75 \times 2^{16}$.

B E C0 00 00
1011 1110 1100 0000 0000 0000 0000 0000
S Exp
negative
exp = 7D → base ten 125
mantissa = 1.1 ← fixed point binary
$= 11 \cdot 2^{-1}$

actual exponent
$125-127=-2$

$V = -(11 \times 2^{-1})(2^{-2})$
$-3 \times 2^{-3} = -\frac{3}{8} = -.375$

16) Repeated division by the number base is primarily useful for what purpose?

a) Converting a number from base-10 to another base.
b) Converting a number between two non-decimal number bases.
c) Converting a number from another base to base-10.
d) Determining how many decimal digits a number has.

17) Repeated multiplication by the number base is primarily useful for what purpose?

a) Converting a number from base-10 to another base.
b) Converting a number between two non-decimal number bases.
c) Converting a number from another base to base-10.
d) Determining how many decimal digits a number has.

18) How is zero represented in an IEEE-754 standard floating point representation?

a) As a pattern of all zeros.
b) Zero cannot be exactly represented because of the implied leading one in the mantissa.
c) As a pattern with a one as the second bit from the left.
d) As a pattern of all zeros except for a leading one.

19) In an IEEE-754 representation, why does an exponent consisting of all zeros and an exponent consisting of all zeros except for a trailing one represent the same exponent value?

a) To avoid a gap as the representation moves from a normalized to a non-normalized interpretation.
b) To increase the static range of the representation.
c) So that zero may be exactly represented.
d) It doesn't.

20) Given a bit pattern representing a floating point value in the standard IEEE format, how can the negative of that value be represented?

a) By taking the two's complement of the mantissa.
b) By inverting the sign bit and all bits of the mantissa.
c) By treating the entire pattern as though it were an integer and taking the two's complement of it.
d) By inverting the sign bit.

11:30

Short Answer (2 pts each)
Questions 16-20 refer to the speed of light in a vacuum which is exactly c = 299,792,458 m/s.

21) How many digits are required to express this in base 7?

$7^{N-1} \le c < 7^{N}$

$(N-1)\ln(7) \le \ln(c)$

$N \le \frac{\ln(c)}{\ln(7)} + 1 = 10.03 + 1 = 11.03$

$N = 11$

11

22) What is the smallest base that could express this value in six digits or less?

$B^{N-1} \le c < B^{N}$ $N = 6$

$c < B^6$

$\frac{\ln(c)}{6} < \ln(B)$

$\ln(B) > \frac{\ln(c)}{6}$

$B > e^{\left(\frac{\ln(c)}{6}\right)} = 25.86$

$B = 26$

26

23) How many bits are required to express this value?

$2^{N-1} \le c < 2^{N}$

$\ln(c) < N \cdot \ln(2)$

$N > \frac{\ln(c)}{\ln(2)} = 28.2$

$N = 29$

29

24) How many digits would the Babylonians have used in their base-60 numbering system?

$c < 60^{N}$

$\frac{\ln(c)}{\ln(60)} < N \Rightarrow N > 4.76$

$N = 5$

5

25) How many hexadecimal digits are required to represent c^2 (the square of the speed of light)?

$c^2 < 16^{N}$

$2\ln(c) < N\ln(16)$

$N > 2 \cdot \frac{\ln(c)}{\ln(16)} = 14.07$

$N = 15$

15

18:49

Longer Answer (10 pts each)

26) Fill in the following table with the decimal value represented by each bit patterns using the representation at the top of the column.

PATTERN	Pure Binary	Signed Binary	Offset Binary	One's Comp	Two's Compl
000	0	0	-4	0	0
001	1	1	-3	1	1
010	2	2	-2	2	2
011	3	3	-1	3	3
100	4	-0	0	-3	-4
101	5	-1	1	-2	-3
110	6	-2	2	-1	-2
111	7	-3	3	-0	-1

27) Performing all computations directly in hexadecimal, write the result of 0xFADE divided by 0x0BAD in quotient-remainder form (i.e., the quotient is an integer).

$\boxed{15 \text{ r } 5AD}$

BAD) FADE
- BAD↓
400E
3A61
5AD

$\rightarrow \approx \frac{\cancel{4000}}{\cancel{C00}} = C\overline{)40}$ 5, 3C

TRY 5: BAD × 5 = 3A61

28) (10 pts) A particular number is represented using two bytes in signed binary, offset binary, and two's compliment format. In no particular order, the three representations yield values of 0xA710, 0x83F0, and 0x58F0. What is the original base-10 number?

SIGN BIT

value	SIGNED	OFFSET	TWO'S
>0	0	1	0
<0	1	0	1

A71Ø → SIGN BIT = 1
83FØ → SIGN BIT = 1 } NEGATIVE #
58FØ → SIGN BIT = 0 ← OFFSET BINARY

OFFSET BINARY:

VALUE = REPRESENTATION − OFFSET

OFFSET = 8000

−VALUE = OFFSET − REPRESENTATION

$$
\begin{array}{r}
= \; 8000 \\
- \; 58F0 \\
\hline
2710
\end{array}
$$

VALUE = −0x2710

$$
\begin{array}{r}
2 \\
\times 16 \\
\hline
32 \\
+7 \\
\hline
39 \\
\times 16 \\
\hline
234 \\
39 \\
\hline
624 \\
+1 \\
\hline
625 \\
\times 16 \\
\hline
3750 \\
625 \\
\hline
10000 \\
+0
\end{array}
$$

$\boxed{-10000}$

29) (10 pts) Write a complete C function called log_b() that takes two floating point arguments. The function returns the logarithm of the first number using the second number as the base. The base is assumed to be strictly positive.

$1000 = 10^3$ ← $\log_{10}(1000)$

x ↑ base ↑

$$x = b^y$$

$$\ln(x) = y(\ln(b))$$

$$y = \frac{\ln(x)}{\ln(b)}$$

```
#include <math.h> /* log() */

double log_b(double x, double b)
{
    return log(x)/log(b);
}
```

(29)

36:08

30) (10 pts) Without using the pow() function, write a complete C function called YrootofX() that takes two floating point arguments. The function returns the yth root of x where x is the first argument (assumed to be strictly positive) and y is the second argument (assumed to be non-zero).

Example: z = YrootofX(64, 3); /* Should store the value 4 in the variable z */

root

$4^{3} = 64$

y x

$y^{root} = x$

$root \cdot \ln(y) = \ln(x)$

$\ln(y) = \frac{\ln(x)}{root}$

$y = \exp\left(\frac{\ln(x)}{root}\right)$

```c
#include<math.h> /* log(), exp() */

double YrootofX(double x, double root)
{
    return( exp(log(x)/root));
}
```

(30)

39:30

31) EXTRA CREDIT #1: (10 pts)
A sign board has six tri-color lights. Each light can display one of three colors – red, green, or blue. The value of each color is as follows: red = 0; green = 1; blue = 2.

a) How many different values can be represented by this sign board? 729

$3^6 = 729$

b) If the lights represent an unsigned integer according to a conventional positional numbering system, what decimal value is represented by the following configuration of lights? 587

LIGHT 5	LIGHT 4	LIGHT 3	LIGHT 2	LIGHT 1	LIGHT 0
BLUE	GREEN	RED	BLUE	RED	BLUE
2	1	0	2	0	2

2×3=6 → 6+1 = 7, ×3 = 21 → +0 = 21, ×3 = 63 → +2 = 65, ×3 = 195 → +0 = 195, ×3 = 585 → +2 = 587

c) Circle the color of each light if the decimal value of the number being displayed 140.

LIGHT 5	LIGHT 4	LIGHT 3	LIGHT 2	LIGHT 1	LIGHT 0
R G B	R G B	R G B	R G B	R G B	R G B

3 | 140
3 | 46 r2
3 | 15 r1
3 | 5 r0
3 | 1 r2
0 r1

012012

d) Using the same reasoning that led to 2's compliment for representing signed values in binary, what color would each light be if –140 is displayed using 3's compliment?

LIGHT 5	LIGHT 4	LIGHT 3	LIGHT 2	LIGHT 1	LIGHT 0
R G B	R G B	R G B	R G B	R G B	R G B

REASONING: $3^6 \Leftrightarrow 0$

$(A)+(-A) = 0 = 3^6$

$-A = 3^6 - A$

$-140 = 729 - 140 = 589$

3 | 589
3 | 196 r1
3 | 65 r1
3 | 21 r2
3 | 7 r0
3 | 2 r1
0 r2

210211

e) Using the same reasoning that led to offset binary for representing signed values in binary, what color would each light be for the value zero?

LIGHT 5	LIGHT 4	LIGHT 3	LIGHT 2	LIGHT 1	LIGHT 0
R G B	R G B	R G B	R G B	R G B	R G B

REASONING: MIDWAY IS ZERO

0 | 000000 | -364 (364 CODES)

364 | ? | = 0 (1 CODE)

(364 CODES)

728 | 222222 | +364

PURE PATTERN OFFSET

729 CODES

ZERO IN OFFSET REPRESENTATION HAS SAME PATTERN AS 364 IN PURE REPRESENTATION

3 | 364
3 | 121 r1
3 | 40 r1
3 | 13 r1
3 | 4 r1
3 | 1 r1
0 r1

111111

EXTRA CREDIT #2 (10 pts)

For large values of n, the factorial of n can be closely approximated by the following equation:

$$n! = \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$$

For instance, for n = 10 the above equation, known as the Stirling Formula, yields a value that is within 1% of the correct answer.

(5 pts) Write a complete C function called fact_digits() that takes a double n as the argument and returns a floating point (double) value that is the number of digits in n!. Your function needs to be able to accept values of n for which n! would be far too large to accommodate even in a variable of long double. For instance, the user might wish to use your function to discover what the first value of n! is that has more than one trillion (10^{12}) digits. A long double, on most compilers, can't even represent a number with 15,000 digits.

Hint: How many digits are in 12345? What is the base ten log of 12345?

$\log_{10}(12345) = 4.09149$

digits = floor $(\log_{10}(12345) + 1)$

(5 pts) Within 1%, what is the first value of n for which n! has more than a thousand digits?

Hint: You can do this reasonably quickly on your calculator - but be sure to be finished with the rest of the exam first. Show your work for full credit.

How many digits in y? digits = floor $(\log_{10}(y) + 1)$

$\log_{10}(y!) = 0.5 \cdot \log_{10}(\cancel{\sqrt{2\pi n}}\ 2\pi n) + n \log_{10}(n/e)$

```c
#include <math.h> /* log10(), floor() exp() */
#define PI (3.1415926)
double fact_digits(double n)
{
    double digits;
    digits = n* log10(n/exp(1));
    digits += 0.5* log10(2.0*PI*n);
    return floor(digits+1);
}
```

PART I

first pass:

$n \cdot \log_{10}(n/e) = 1000$

n	$n \cdot \log_{10}(n/e)$
1000	2566
500	1132

$500 \cdot \frac{1000}{1132} = 442$

442	977
452	1003.8

450

FULL EQUATION

		FOR 451	FOR 450
$0.5 \log_{10}(2\pi n) =$	1.7218	1.726	1.725
$n \cdot \log_{10}(n/e) =$	977.318	1001.166	998.5
	979	1002.9	1000.23

21 DIGITS SHORT:

2 DIGITS

450 → 1001 digits

~~449~~ → 998 digits

1:06:21