Implementation of a CDMA Receiver with Multiple-Access Noise Rejection

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ABSTRACT

A DS-CDMA receiver is typically implemented as a matched filter. Such a receiver, while optimum in additive white gaussian noise, may yield poor performance when multiple-access noise is the dominant interference. Previously proposed multiple-user receivers greatly improve performance in this situation, but quickly become prohibitively complex for systems of realistic size. Here we propose a receiver which takes advantage of the colored power spectrum of the multiple-access noise to reject it. The receiver is simple to implement, can run at high rates, and can be tuned adaptively. It is designed to maximize signal-to-noise ratio, but is also shown to yield a substantial improvement over the conventional receiver in average probability of error.

INTRODUCTION

It is well known that a matched filter is an optimal receiver in additive white gaussian noise (AWGN). When multiple DS-CDMA users access the same AWGN channel, each receiver also sees non-gaussian, non-white, multiple-access interference, so a matched receiver is no longer optimal. Previous papers have proposed alternate receiver structures. Most of these (e.g. [1]) are based on locking and despreading many (or all) of the CDMA signals simultaneously. This may be highly impractical, so instead we concentrate on receivers which operate under two main constraints: they base decisions on the received waveform over approximately one bit time, and they do not require knowledge of any spreading sequences other than that of the single user of interest.

Here we investigate the use of a transversal filter, with taps spaced at a *fraction* of the chip time, to reject multiple-access noise. This paper extends and complements results presented in [2]. The difference is that the receiver here is more amenable to implementation at a high chip rate, and can be tuned adaptively without performing any matrix inversion.

Suppose that a CDMA system with M users is modeled as in Figure 1. The bit time is denoted by T, the processing gain by N, and the chip time by $T_c \equiv \frac{T}{N}$. All spreading waveforms are rectangular and binary valued. The spreading codes for users 2 through M are modeled as independent random binary sequences at rate $1/T_c$, but the code for user 1 is some fixed N chip sequence. For users 2 through M, the data on the random binary sequence is irrelevant and is not included in the model. The θ_i and τ_i are all assumed independent and uniformly distributed over $[0, 2\pi]$ and $[0, T_c]$, respectively. The $n_w(t)$ is AWGN of two sided PSD $\frac{\eta_o}{2}$. There is no RF bandpass filtering in the model. Perfect power control is assumed, so that all M CDMA signal amplitudes are the same at the receiver. E_b is defined as $A^2T/2$.

The statistics r_i are integrals of the received waveform over *fractions* of a chip interval, $T_q \equiv \frac{T_c}{N_q}$, for some integer N_q. These are referred to as subchips. We assume throughout that

$$\omega T_q = n2\pi \tag{1}$$

for some integer n, with ω the carrier frequency from Figure 1.

The box labelled $A(z)$ is a two sided FIR filter with K taps per side, of transfer function

$$A(z) = \sum_{n=-K}^{K} \alpha_n z^{-n} \tag{2}$$

Note that if $\alpha_n = \delta_n$, where δ_n is the Kroneker delta function, then the receiver above is the same as a conventional receiver.

In discrete notation, c_i is the value of $c^1(t)$ for the interval $(i-1)T_q < t < iT_q$. Thus, the series c_i consists of the chips of $c^1(t)$ each repeated N_q times. All other discrete quantities are also defined on a T_q time base. The notation below assumes that we are receiving the bit transmitted in the interval $[0, T]$, and that $d^1(t) = +1$ in this interval.

Using these assumptions and Figure 1,

$$r_i \equiv \int_{(i-1)T_q}^{iT_q} r(t) 2\cos(\omega t) dt$$

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$$= AT_q c_i + \sum_{m=2}^{M} I_i^m + n_i, \quad (3)$$

where

$$I_i^m = A\cos\theta_m \int_{(i-1)T_q}^{iT_q} c^m(t-\tau_m)dt,$$

and

$$n_i = \int_{(i-1)T_q}^{iT_q} n_w(t) 2\cos(\omega t)dt.$$

To determine signal-to-noise ratio, it is necessary to define the autocorrelation of the noise, including white and multiple-access noise. The assumptions above make the noise a stationary process, so this autocorrelation can be written as

$$\phi_{i-j} = E[J_i\ J_j], \quad (4)$$

where

$$J_i \equiv n_i + \sum_{m=2}^{M} I_i^m. \quad (5)$$

The evaluation of this series for rectangular spreading pulses is detailed in [2]. The technique proposed here could be applied to other chip waveforms simply by evaluating the appropriate autocorrelation function (4).

The decision statistic is given by

$$G_1 = \sum_{i=1}^{NN_q} c_i \sum_{j=-K}^{K} \alpha_j r_{i-j} \quad (6)$$

$$= \sum_{j=-(K-1)}^{NN_q+K} r_j q_j, \quad (7)$$

where

$$q_j = \sum_{i=-K}^{K} \alpha_i c_{j-i} p_{j-i} \quad (8)$$

and p_i is the series which is one for $i \in [1 \cdots NNq]$ and zero elsewhere.

Figure 1 uses a discrete time filter only for mathematical simplicity. Under the assumption of (1), this model is equivalent to passing the received RF or IF waveform directly through a transversal filter with tap spacing T_q, before going to a conventional CDMA receiver. This latter form could be feasibly implemented even at a high chip rate.

TAP WEIGHTS WHICH MAXIMIZE SNR

Consider the model of Figure 1, where user 1, instead of transmitting a stream of bits, transmits only one bit, i.e.

$$d^1(t) = \pm p_T(t), \quad (9)$$

where $p_T(t)$ is 1 for $t \in [0,\ T]$ and 0 elsewhere.

Assuming (9), it is a common result of detection theory [3] that the filter which maximizes SNR satisfies

$$A(e^{j\omega}) = \frac{1}{\Phi(e^{j\omega})}, \quad (10)$$

where $A(z)$ is given by (2) with infinite K, and $\Phi(z)$ is the z-transform of the noise autocorrelation defined by (4). Thus, $\Phi(e^{j\omega})$ is the discrete time noise power spectral density (PSD). Applying (10),

$$\alpha_n = \frac{1}{2\pi}\int_0^{2\pi} \frac{1}{\Phi(e^{j\omega})} e^{j\omega n} d\omega. \quad (11)$$

This α_n is a real, even series. It is of infinite duration in general, and so cannot be implemented exactly as a transversal filter of finite size K. However, considering this case provides a useful insight: when (10) is satisfied, Figure 1 represents the optimal receiver for detecting $\pm c_i p_i$ in colored *gaussian* noise of PSD $\Phi(e^{j\omega})$, using an infinite observation interval [3]. Thus, the receiver proposed here can be viewed as resulting from approximating J_i as a colored *gaussian* process of the same PSD. By the central limit theorem, such an approximation should become more accurate the larger M is. It is shown in [3] that when (10) is satisfied, Figure 1 can be thought of as a noise whitening filter followed by a matched filter.

Now consider the case in which the filter is constrained to K taps per side, as in (2). This result can be expressed concisely in vector notation. The sequence q_i defined by (8) is non-zero for $j \in [-(K-1),\ NN_q+K]$. So $\bar{q}$ is defined as the NN_q+2K by 1 vector $[q_{-(K-1)}\ q_{-(K-2)}\ \cdots\ q_{NN_q+K}]^T$. The vectors $\bar{r}$ and $\bar{J}$ are similarly defined as the same span of r_i and J_i, respectively. These series are given by (3) and (5). $\bar{\bar{\Phi}} \equiv E[\bar{J}\bar{J}^T]$ is the covariance matrix of the noise process. The vector $\bar{\alpha}$ is defined as $[\alpha_{-K}\ \cdots\ \alpha_K]^T$. $\bar{0}_j$ denotes a column vector of j zeros. The vector $\bar{\gamma}_j$ is defined as $[\bar{0}^T_{(2K-j)}\ , c_1,\ c_2,\ \cdots,\ c_{NN_q},\ \bar{0}^T_j]^T$. The matrix $\bar{\bar{C}}$ is defined as $[\bar{\gamma}_0,\ \bar{\gamma}_1,\ \cdots,\ \bar{\gamma}_{2K}]$. This definition allows $\bar{q}$ to be expressed concisely as $\bar{q} = \bar{\bar{C}}\bar{\alpha}$. The signal vector $\bar{c}$, under the assumption of (9), is defined as $\bar{\gamma}_K$. In this vector notation, (7) can be rewritten, using (3) and (5), as

$$G_1 = \bar{r}^T\bar{q} = A\bar{c}^T\bar{q} + \bar{J}^T\bar{q} \quad (12)$$

From (12), the signal-to-noise ratio can be written as

$$SNR_{\bar{\alpha}} = \frac{\left(A\bar{c}^{T}\bar{q}\right)^{2}}{\bar{q}^{T}\bar{\bar{\Phi}}\bar{q}} = \frac{A^{2}\left(\bar{c}^{T}\bar{\bar{C}}\bar{\alpha}\right)^{2}}{\bar{\alpha}^{T}\bar{\bar{C}}^{T}\bar{\bar{\Phi}}\bar{\bar{C}}\bar{\alpha}}. \tag{13}$$

It can be shown that the $\bar{\alpha}$ which maximizes (13) is the solution to

$$\bar{\bar{M}}\bar{\alpha} = \bar{v}, \tag{14}$$

where

$$\bar{\bar{M}} = \bar{\bar{C}}^{T}\bar{\bar{\Phi}}\bar{\bar{C}}$$

and

$$\bar{v} = \bar{c}^{T}\bar{\bar{C}}.$$

This yields the optimal tap weights under the criterion of maximum SNR.

This derivation is based on (9). In practice, $d^1(t)$ would be a long stream of data bits. But it is reasonable to assume that $K \ll NN_q$, otherwise the filter $A(z)$ would be too large to implement. If $K \ll NN_q$, then the ISI introduced by $A(z)$ into the final decision statistic will not be significant. This statement is quantitatively justified in [4].

APPROXIMATE TAP WEIGHTS

It is well known [5] that the common two-sided LMS filter observing the J_i process also yields the tap weights of equation (11) for infinite K. This suggests that perhaps the LMS filter might be substituted for (14) for finite K as well. In fact, the two are similar for $K \gg N_q$, but can become significantly different for smaller K [4].

The solution to (14) takes into account the exact spreading code used, in the form of $\bar{c}$. It is shown in [4] that a very close approximation to (14) can be found by replacing the actual c_i series with

$$c_i = \begin{cases} 1, & \text{for } i \in [1,\ N_q], \\ 0, & \text{elsewhere.} \end{cases} \tag{15}$$

and *then* solving (14). This filter is called the "one-chip" filter, since (15) corresponds to the shape of a single chip. It can be shown that the appearance of $\bar{c}$ in (14) can be expressed entirely in terms of the aperiodic autocorrelation of $\bar{c}$. Given this, it is not surprising that the one-chip filter gives an accurate approximation of (14), since the autocorrelation of any reasonable spreading vector $\bar{c}$ is likely to be similar in shape to the autocorrelation of (15).

The LMS and one-chip filters are similar, and in fact it can be shown that the LMS filter is a special case of the one-chip filter which results from substituting $N_q = 1$ in (15). Similar to the LMS filter, the one-chip filter can be tuned by various adaptive algorithms without solving the matrix equation (14). This is one of the main advantages of this implementation over that presented in [2]. Details of an adaptive structure are shown in [4].

An adaptive filter would ideally use, as input, the noise process J_i of (5). However, as long as $M \gg 1$, the autocorrelation of the r_i process is not significantly different from that of J_i. Using the r_i process directly to tune the adaptive filter is simpler and introduces very little error, as shown in the next section.

RESULTS

The figures refer to the receiver of Figure 1 depending on the value of $\bar{\alpha}$, the tap vector of the filter $A(z)$, as follows:

- Receiver I : $\alpha_n = \delta_n$. This is a conventional (unfiltered) CDMA receiver.
- Receiver II : $\bar{\alpha}$ set by (14), with $\bar{c}$ being the spreading sequence of user 1.
- Receiver III : $\bar{\alpha}$ set by LMS criterion, as in [5].
- Receiver IV : $\bar{\alpha}$ set by (14), with $\bar{c}$ being the one-chip sequence of (15).
- Receiver V : $\bar{\alpha}$ set by the adaptive filter described in [4], adapting to r_i process instead of J_i. The expected steady-state $\bar{\alpha}$ is used, so no tap adjustment error is present.

Figures 2, 3, and 4 plot P_e^g for a system of $N = 128$, $M = 39$, and $c^1(t)$ a particular 128 chip sequence listed in [4]. P_e^g denotes the gaussian approximation of the probability of error, which is $\phi(-\sqrt{SNR})$, where

$$\phi(x) \equiv \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{t^2/2}dt,$$

and SNR is the signal squared over the noise variance in the final decision statistic. These values are for single bit reception, as in (9). For reference, these figures also plot P_e^g for antipodal signalling in AWGN, which can be thought of as receiver I in the absence of multiple-access noise.

Figures 2 and 3 illustrate the effect of N_q and K. Diminishing returns are generally seen as either increases. However, Figure 2 shows jumps in SNR when K increases to 1 more than a multiple of N_q. This is explained further in [4]. A lower bound on P_e^g as N_q and K increase infinitely has been derived by A. Monk [4], and is only slightly below the $N_q = 8$ curve in Figure 3.

Figure 4 plots the P_e^g resulting from several filters for $N_q = 4$ and $K = 4$. Receiver II has the lowest P_e^g since it maximizes SNR. The LMS filter gives similar performance at low E_b/η_o, but suddenly worsens at higher E_b/η_o. Receiver V yields a P_e^g so close to that of receiver II that the two cannot be distinguished on the graph. Thus, the errors introduced by adapting to r_i instead of J_i, and by using the one-chip filter as opposed to the filter of (14), are negligible for this choice of parameters.

All the previous derivations and results have been based on signal-to-noise ratio, which is a helpful performance measure, but is not directly related to average probability of error when the interference is non-gaussian. The accuracy of the gaussian approximation for error probability in CDMA systems is discussed in [6].

Figure 5 plots the average probability of error, denoted by P_e, calculated by the characteristic function method of [7]. It is believed that the numerical results are accurate to within the resolution of the plots. The system analyzed is that of 9 asynchronous users, using processing gain 31, each employing Gold spreading codes of period 31. In other respects the model used was similar to that of Figure 1. $N_q = 4$, $K = 5$, and (9) is not assumed, so the model accounts for filter-induced ISI. P_e is plotted for one of the users. Each curve corresponds to a fixed $\bar{\alpha}$. If $\bar{\alpha}$ were correctly tuned with varying E_b/η_o, then P_e would approximately follow the minimum of all such curves. It can be seen that P_e is substantially improved by the transversal filter receiver, and the performance is comparable to that found in [2].

CONCLUSIONS

It has been shown that for rectangular spreading pulses, the transversal filter receiver of Figure 1 can achieve a large improvement in signal-to-noise ratio over the conventional receiver. This also translates into an improvement in average probability of error, even for a system of small M, where the gaussian approximation of error probability is less accurate. The receiver could be practically implemented using a transversal filter of tap spacing T_c/N_q directly on the RF or IF received waveform.

Since the technique presented here depends on multiple-access noise whitening, a flat-spectrum pulse shape cannot benefit from it. For example, using raised-cosine spectrum pulses, if there is 0% excess bandwidth, the noise spectrum is flat and there is no gain in filtering. As excess bandwidth increases, more benefit is seen. Flat-spectrum pulses are the most spectrally efficient [8], but, by using a filter as described here, the SNR for non-flat-spectrum pulses can be made to approach that of flat-spectrum pulses as multiple-access noise becomes large compared to thermal noise. Using a pulse shape with a rounded spectrum, with filtering at the receiver to compensate, may offer implementation advantages over attempting to approximate flat-spectrum pulses.

All the analysis presented here can be equivalently phrased in terms of equalization, as opposed to detection. Equalization is typically thought of as compensating for ISI, but it can also compensate for reception in colored noise. Thus, the filter presented here can also be derived as a fractionally-spaced minimum MSE equalizer. All of these points are developed in detail in [4].

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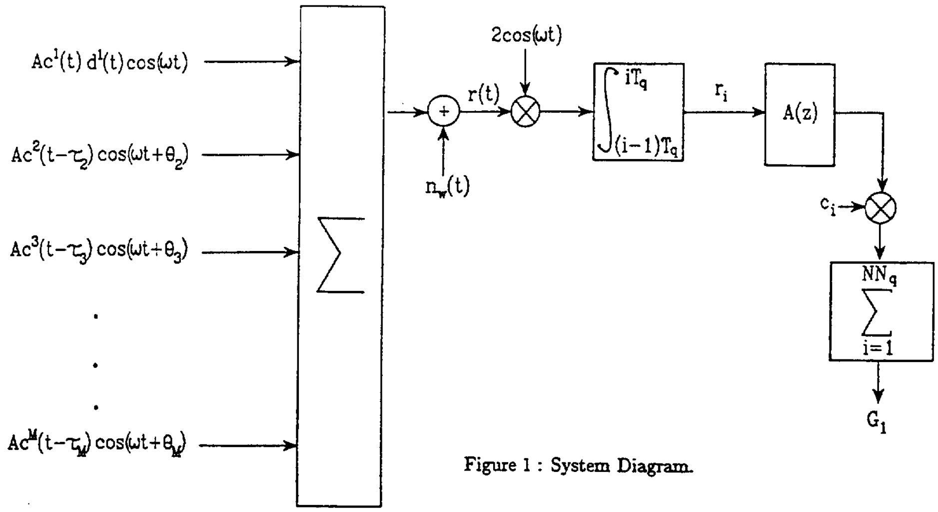

Figure 1 : System Diagram.

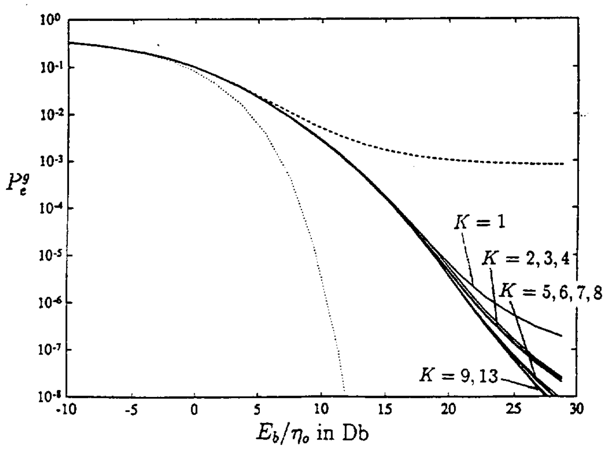

Figure 2 : Dashed line is receiver I. Dotted line is $\phi(-\sqrt{2E_b/\eta_o})$. Solid lines are receiver II for $N_q=4$ with varying K.

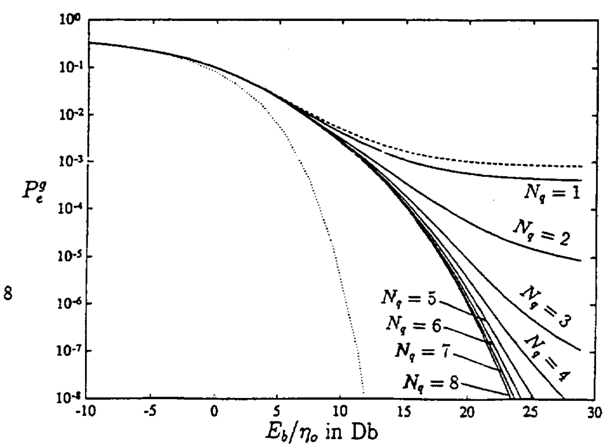

Figure 3 : Dashed line is receiver I. Dotted line is $\phi(-\sqrt{2E_b/\eta_o})$. Solid lines are receiver II, for varying N_q with $K=2N_q+1$.

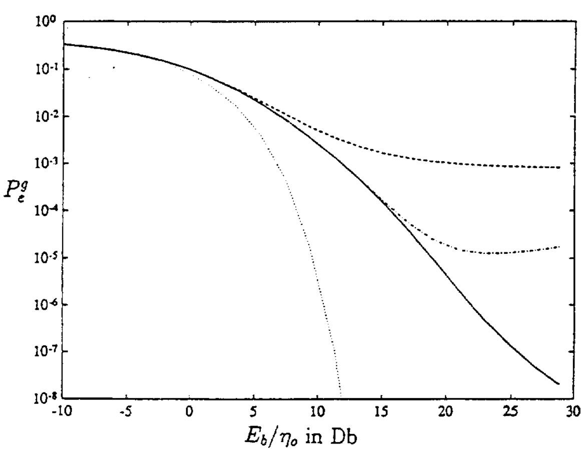

Figure 4 : Dashed line is receiver I. Dotted line is $\phi(-\sqrt{2E_b/\eta_o})$. Dash-Dot line is receiver III. Solid line is receivers II *and* V.

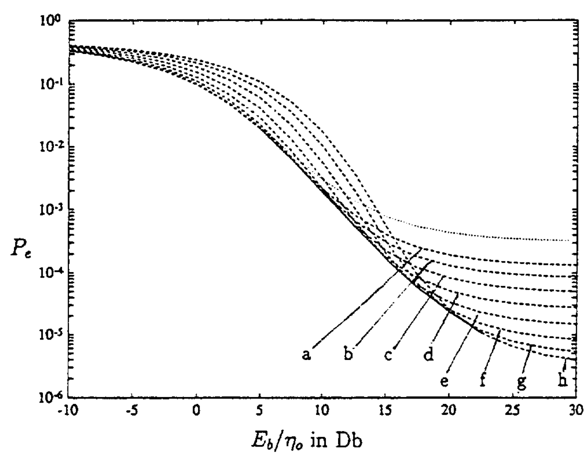

Figure 5 : Dotted line is receiver I. Lines "a" → "h" are for receiver IV with $\bar{\alpha}$ optimized for $E_b/\eta_o = 5$ Db → 22.5 Db in 2.5Db steps.